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PROCEDURES FOR MEASUREMENT IN STRATIFIED GASES. VOLUME II. APPENDICES

A. Zakak, et al

Abcor, Incorporated

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September 1974

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PROCEDURES FOR MEASUREMENT IN STRATIFIED GASES
VOLUME II, APPENDICES

by

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Contract No. 68-02-1306
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Research Triangle Park, North Carolina 27711

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16. ABSTRACT

The report gives results of a program to develop methods for the continuous extraction of representative gas samples from gas streams that exhibit compositional stratification. The program considered available data in the literature, as well as field data generated during the program. Wind tunnel tests and mathematical modeling were used to develop sampling methodologies which are recommended. Data from the literature, as well as program data, indicate that stratification exists, although it is unlikely that gas stratification is as widespread or as severe as particulate stratification. Depending on conditions, two different methods are recommended. The first method involves monitoring the ratio of SO2, NOx, etc. to CO2 at a single location. Then, from the measured fuel flow and chemistry of the process, the mass flow of CO2 can be predicted. The product of the measured ratio and the predicted mass flow of CO2 is the mass flow of the pollutant. Where conditions do not permit using this method, it is recommended that a schedule of manual surveys be conducted followed by installation of a multi-element proportional sampler and gas velocity array.

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Measurement	Data	Stationary Sources	14B
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Stratification			
Wind Tunnels			

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TABLE OF CONTENTS

Appendix	Title	Page
A	Automatic Isokinetic Sampling Systems	1
B	Arrays and Mechanical Traversing Systems	12
C	Diffusion Tube Sampling	19
D	Temperature Stabilized Diffusion Tube	31
E	Tracer Methods	33
F	Thermal Null Probes	35
G	Velocity and Concentration Profiles	37
H-1	The Effect of Jet Impingement on Mixing in a Gas Stream	65
H-2	Jet Mixing in a Duct	84
I	Combustion and Material Balance Calculations	89
J	Computer Program for Analytical Simulation of Procedures for Velocity and Emission Measurements in Stratified Stack Gases (Theory and User Manual)	102

APPENDIX A

AUTOMATIC ISOKINETIC SAMPLING SYSTEMS

Presented in this appendix are a variety of automatic isokinetic sampling instruments. This set is not intended to encompass all the particular conceptual or implemented instruments of this type, but only the devices discovered in our literature survey. However, the instruments presented herein span the range of the general approaches to automatic isokinetic sampling and, by extension, the general approaches to automatic proportional sampling.

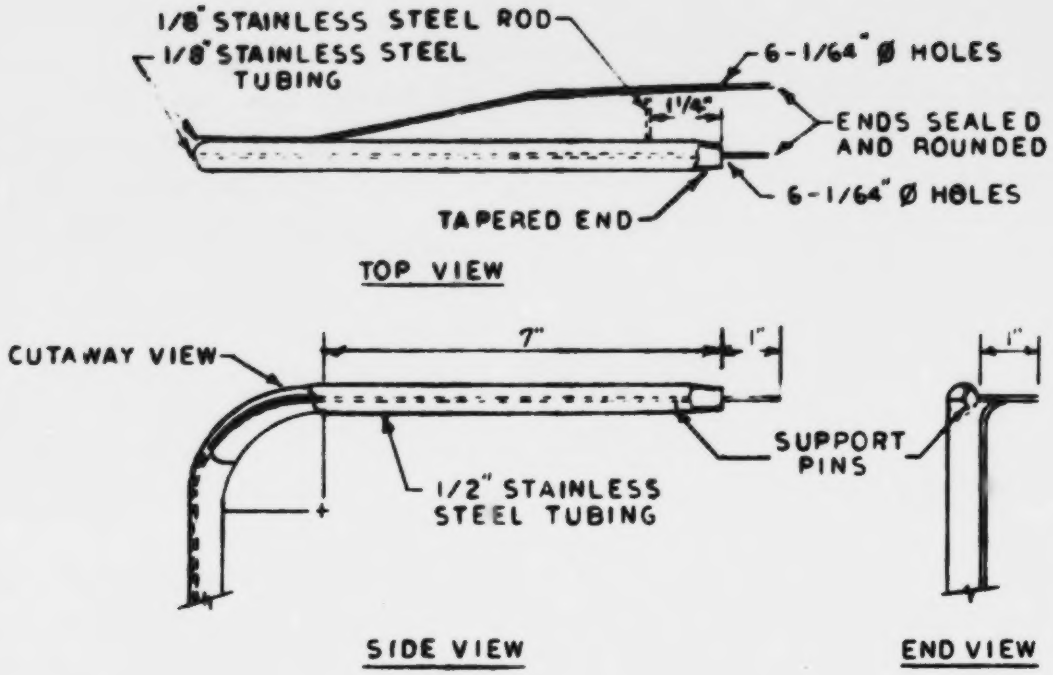

1/8" STAINLESS STEEL ROD
1/8" STAINLESS STEEL TUBING
6-1/64" Ø HOLES
1 1/4"
ENDS SEALED AND ROUNDED
6-1/64" Ø HOLES
TAPERED END
TOP VIEW
CUTAWAY VIEW
7"
1"
SUPPORT PINS
1/2" STAINLESS STEEL TUBING
1"
SIDE VIEW
END VIEW

FIGURE A-1

STATIC PRESSURE NULL PROBE

This is an example of a null balance probe reported by Wilson and Falgout[14*]. The distinct advantage of this particular probe is that the static taps for the sampling probe are in the plane of the orifice, not inside the probe nozzle. When the static ports are inside the nozzle, there are entrance losses which require the probe to be calibrated (see Figure A-2). In this device the static pressure sensing is made in the region just upstream of where entrance losses begin.

(*) References are listed in Vol. I of this report.

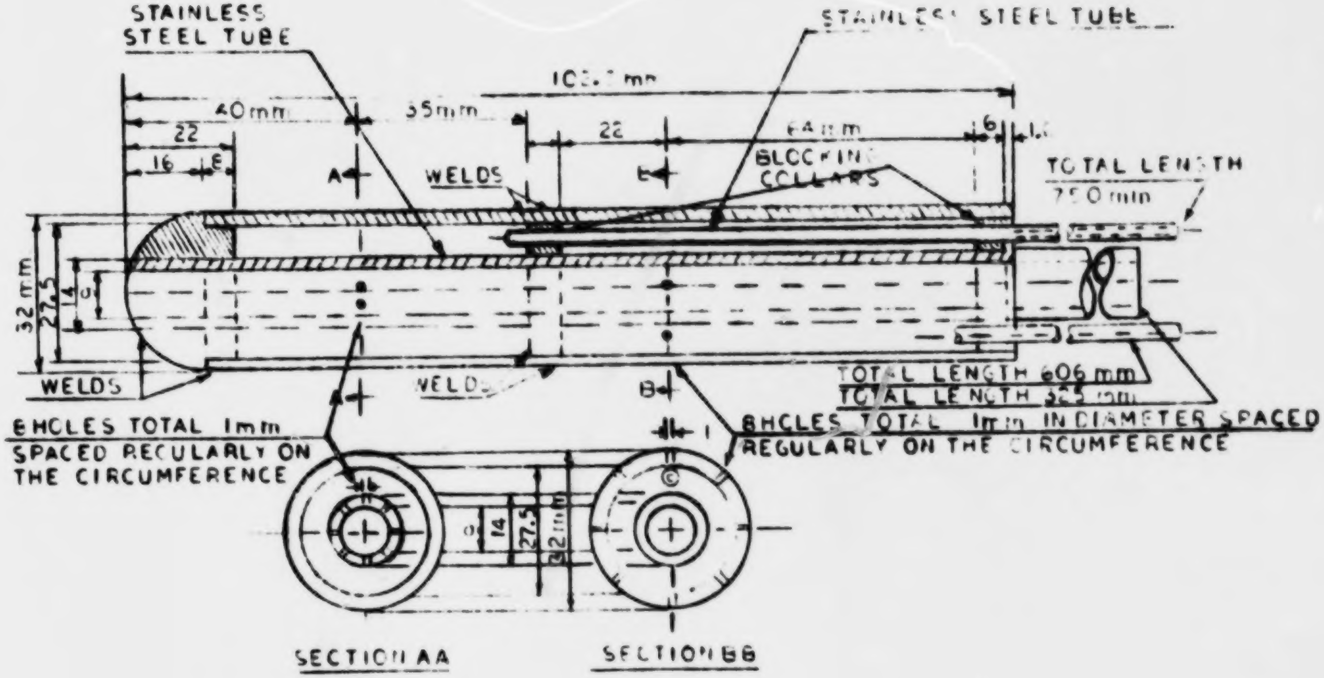
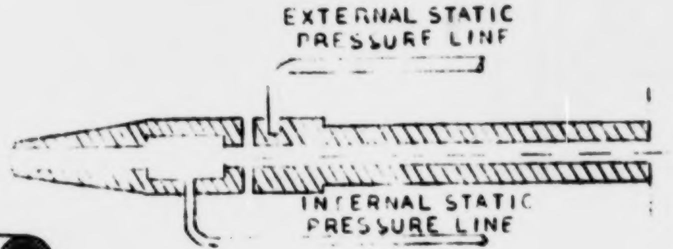

STAINLESS STEEL TUBE
STAINLESS STEEL TUBE
40mm
22
16
8
A
55mm
WELDS
22
B
64 mm
6
BLOCKING COLLARS
TOTAL LENGTH 750 mm
32 mm
27.5
14
8
WELDS
WELDS
B
TOTAL LENGTH 606 mm
TOTAL LENGTH 325 mm
8HOLES TOTAL 1mm SPACED REGULARLY ON THE CIRCUMFERENCE
8HOLES TOTAL 1mm IN DIAMETER SPACED REGULARLY ON THE CIRCUMFERENCE
A
1
8
14
27.5
32mm
SECTION AA
SECTION BB

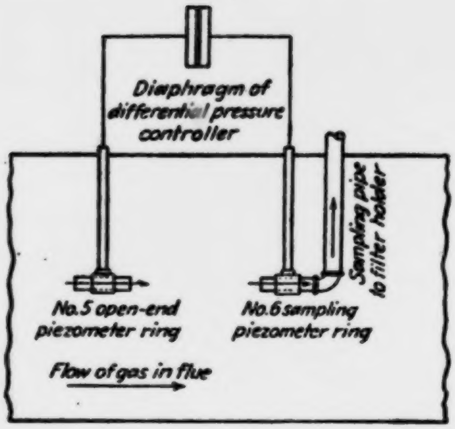
FIGURE A-2

STATIC PRESSURE NULL TYPE PROBES 15

THIS NULL PROBE USES THE
TAPS INSIDE THE NOZZLE
SEE NOTE WITH FIGURE A-1


EXTERNAL STATIC PRESSURE LINE
INTERNAL STATIC PRESSURE LINE

Reproduced from
best available copy.


Diaphragm of differential pressure controller
Sampling pipe to filter holder
No.5 open-end piezometer ring
No.6 sampling piezometer ring
Flow of gas in flue

2-2	Butterfly dampers
3-3	Lead from the Piez ometer rings
4-4	to controllers
5-5	Piezometer rings
6-6	
7-7	Differential pressure recorders
8-8	Differential pressure controllers
9-9	Regulating valves
10	Filter holder
11	Filter fabric
12	Suction system to downstream of sampling pipe

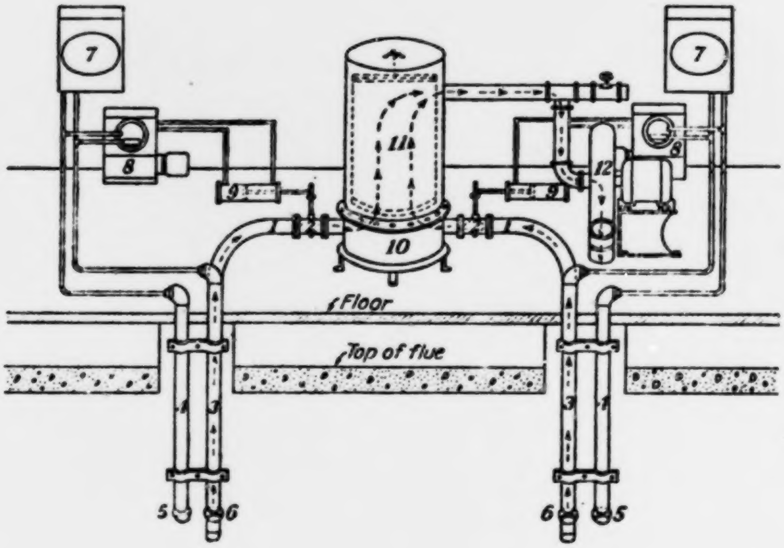

7
7
8
8
9
9
11
12
10
Floor
Top of flue
4
3
3
4
5
6
6
5

FIGURE A-3

AUTOMATIC MULTIPLE SMOKE SAMPLER[16]

This is the only example of a multi-probe isokinetic sampler found.

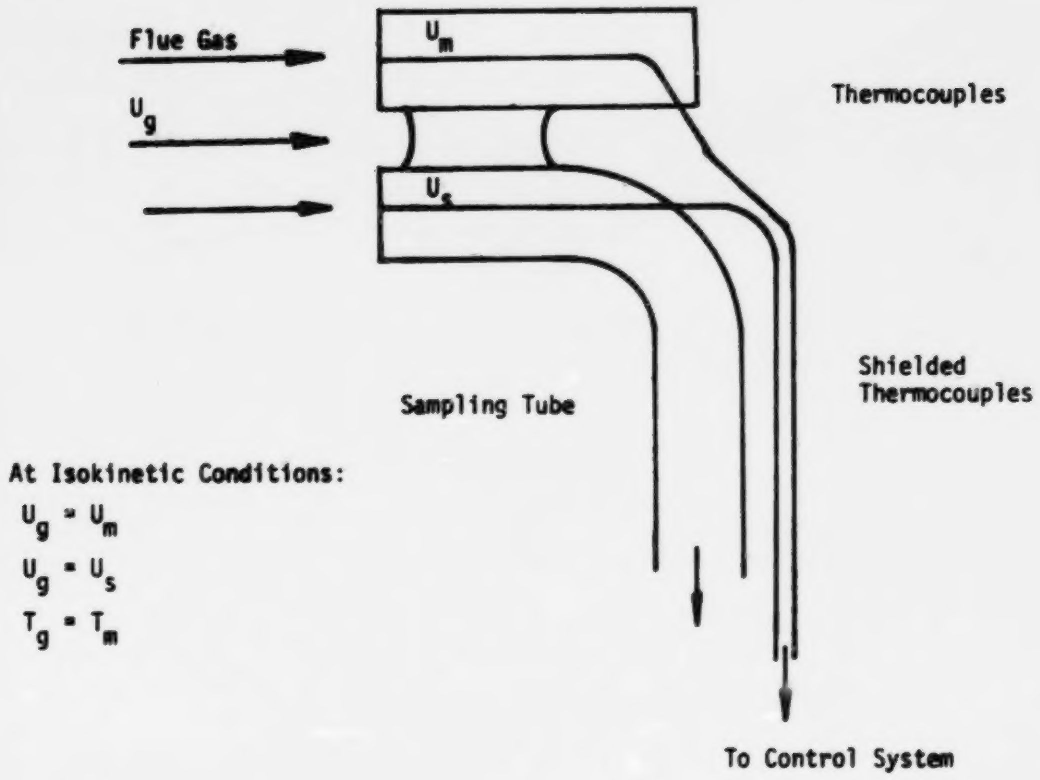
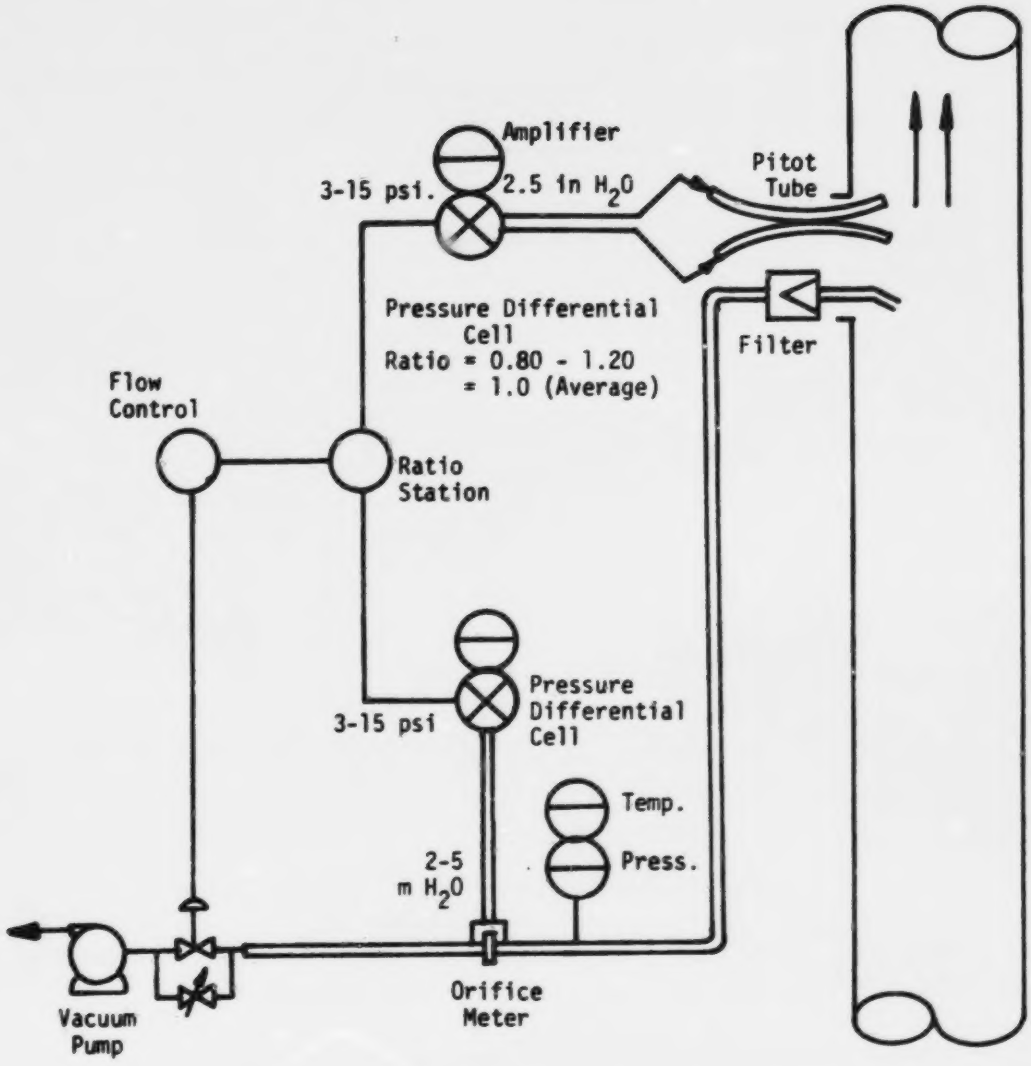

Flue Gas
U_g
U_m
Thermocouples
U_s
Shielded Thermocouples
Sampling Tube
At Isokinetic Conditions:
$U_g = U_m$
$U_g = U_s$
$T_g = T_m$
To Control System

FIGURE A-4

THERMAL NULL TYPE SYSTEM[9, 10, 17]

This is an example of a null type system which makes use of a difference in stagnation temperature to sense velocity (See Appendix F).

Thermal anemometers in place of thermocouples would provide the same information, probably with greater sensitivity at the expense of instrumental complication.



Amplifier
3-15 psi.
2.5 in H_2O
Pitot Tube
Pressure Differential Cell
Ratio = 0.80 - 1.20
= 1.0 (Average)
Filter
Flow Control
Ratio Station
Pressure Differential Cell
3-15 psi
Temp.
Press.
2-5 m H_2O
Vacuum Pump
Orifice Meter

FIGURE A-5

CONTINUOUS SAMPLING RATE ADJUSTMENT BY AUTOMATIC CALCULATION[26]

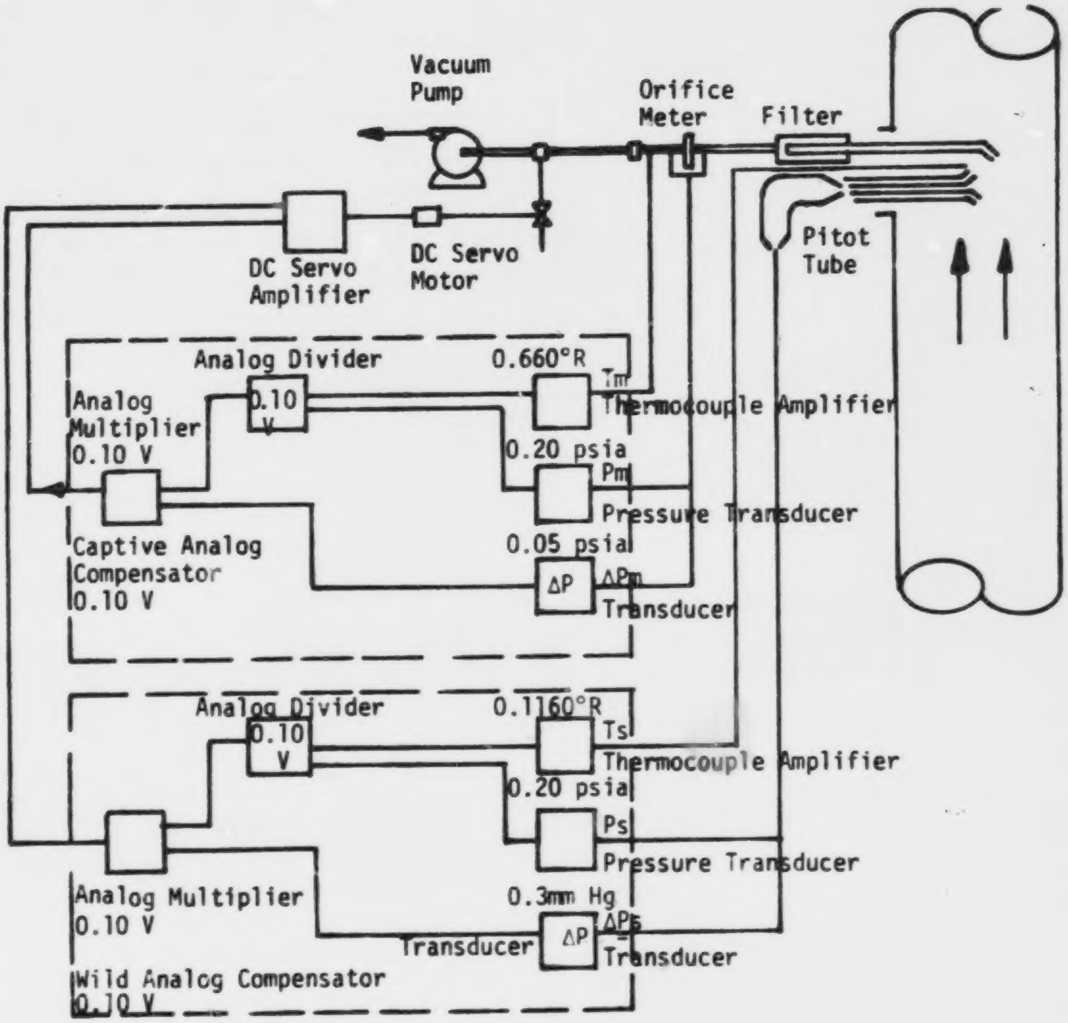

Vacuum Pump
Orifice Meter
Filter
Pitot Tube
DC Servo Amplifier
DC Servo Motor
Analog Divider
0.10 V
0.660°R
Tm
Thermocouple Amplifier
Analog Multiplier
0.10 V
0.20 psia
Pm
Pressure Transducer
Captive Analog Compensator
0.10 V
0.05 psia
ΔP
ΔPm
Transducer
Analog Divider
0.10 V
0.1160°R
Ts
Thermocouple Amplifier
0.20 psia
Ps
Pressure Transducer
Analog Multiplier
0.10 V
0.3mm Hg
Transducer
ΔP
ΔPs
Transducer
Wild Analog Compensator
0.10 V

Figure A-6

CONTINUOUS SAMPLING RATE ADJUSTMENT

BY AUTOMATIC CALCULATION[7]

This system is an improvement over the system shown in Figure A-7 because temperature and pressure for both the stack and sample gas is used in the automatic calculation.

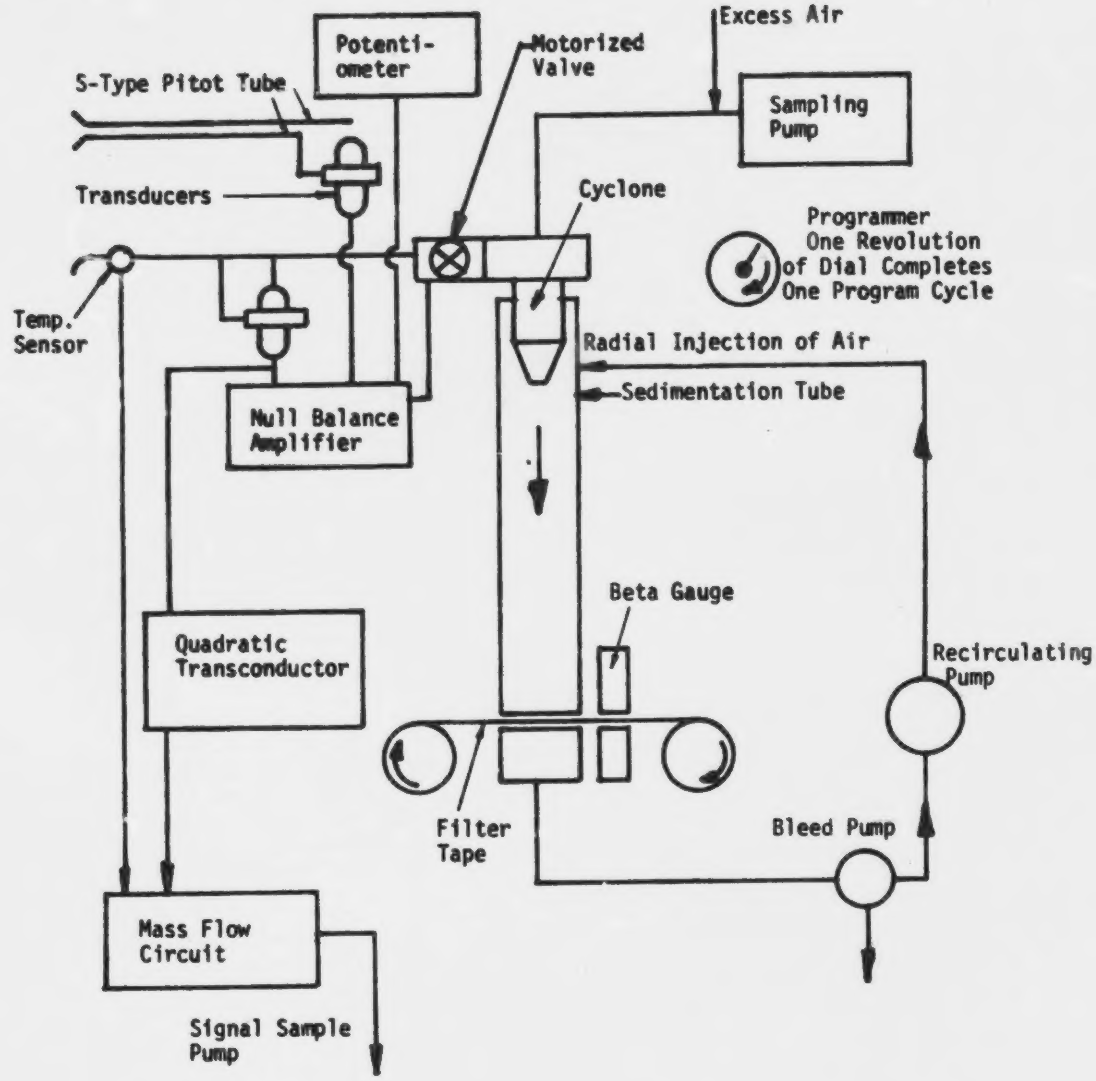

Potentiometer
Motorized Valve
Excess Air
S-Type Pitot Tube
Sampling Pump
Transducers
Cyclone
Programmer One Revolution of Dial Completes One Program Cycle
Temp. Sensor
Radial Injection of Air
Sedimentation Tube
Null Balance Amplifier
Beta Gauge
Quadratic Transconductor
Recirculating Pump
Filter Tape
Bleed Pump
Mass Flow Circuit
Signal Sample Pump

FIGURE A-7

CONTINUOUS SAMPLING RATE ADJUSTMENT BY CALCULATION[19,20]

This system includes a thermistor to compensate for temperature fluctuations in the sampled gas.

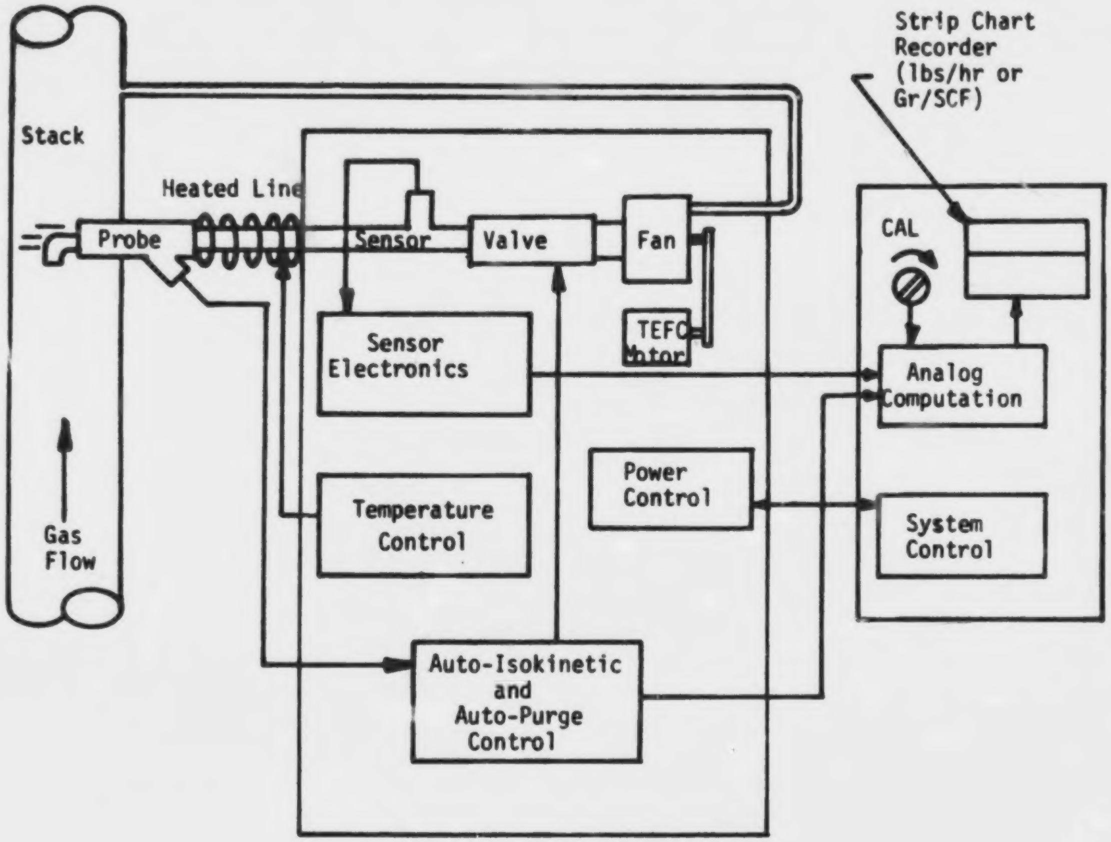

Strip Chart
Recorder
(lbs/hr or
Gr/SCF)
Stack
Heated Line
Probe
Sensor
Valve
Fan
CAL
TEFC
Motor
Sensor
Electronics
Analog
Computation
Power
Control
Temperature
Control
System
Control
Gas
Flow
Auto-Isokinetic
and
Auto-Purge
Control

FIGURE A-8

CONTINUOUS SAMPLING RATE ADJUSTMENT
BY AUTOMATIC CALCULATION[21]

This device has a unique feature in that the sample temperature is automatically held at the same value as the stack temperature, thereby removing the temperature parameter from the gas density calculation.

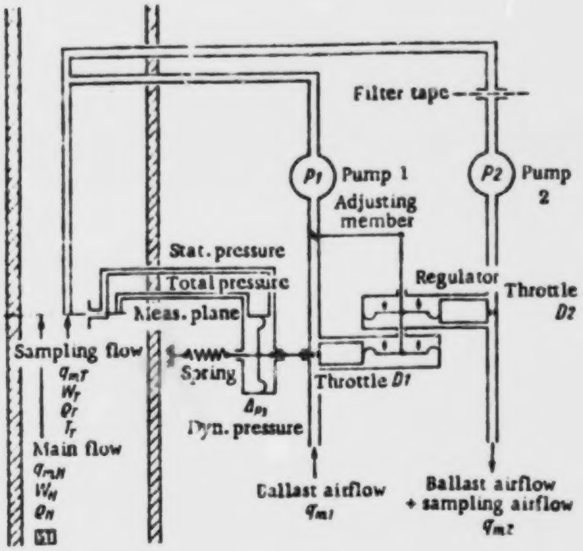

Filter tape
P1
Pump 1
P2
Pump 2
Adjusting member
Stat. pressure
Total pressure
Regulator
Throttle D2
Meas. plane
Sampling flow
q_{mT}
W_T
ϱ_T
T_T
Spring
Throttle D1
Δp_3
Dyn. pressure
Main flow
q_{mN}
W_N
ϱ_N
Ballast airflow
q_{m1}
Ballast airflow + sampling airflow
q_{m2}

FIGURE A-9

CONTINUOUS SAMPLING RATE ADJUSTMENT
BY AUTOMATIC CALCULATION[22]

The unique feature of this device is that the control system is pneumatic, instead of the usual electrical control system.

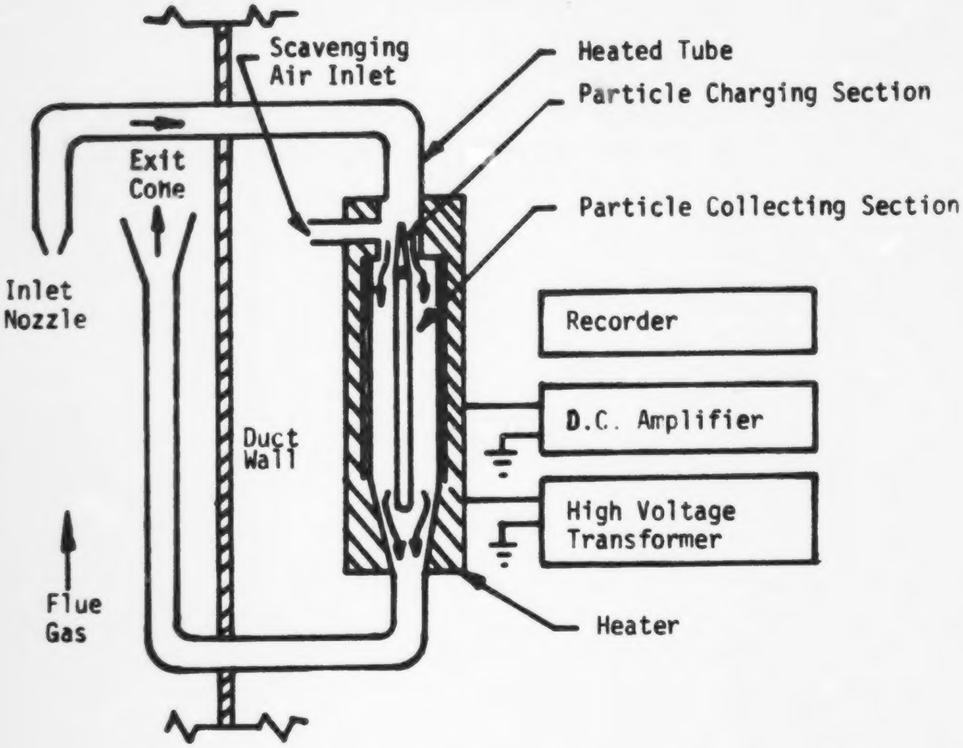

Scavenging
Air Inlet
Heated Tube
Particle Charging Section
Exit
Cone
Particle Collecting Section
Inlet
Nozzle
Recorder
D.C. Amplifier
Duct
Wall
High Voltage
Transformer
Flue
Gas
Heater

FIGURE A-10

PASSIVE, GAS-STREAM DRIVEN AUTOMATIC ISOKINETIC SAMPLER[11,12]

By design of the inlet nozzle and exit cone, the sample flow to the particle collection section will be isokinetic, hence proportional to the gas stream velocity, over a range of operating velocities.

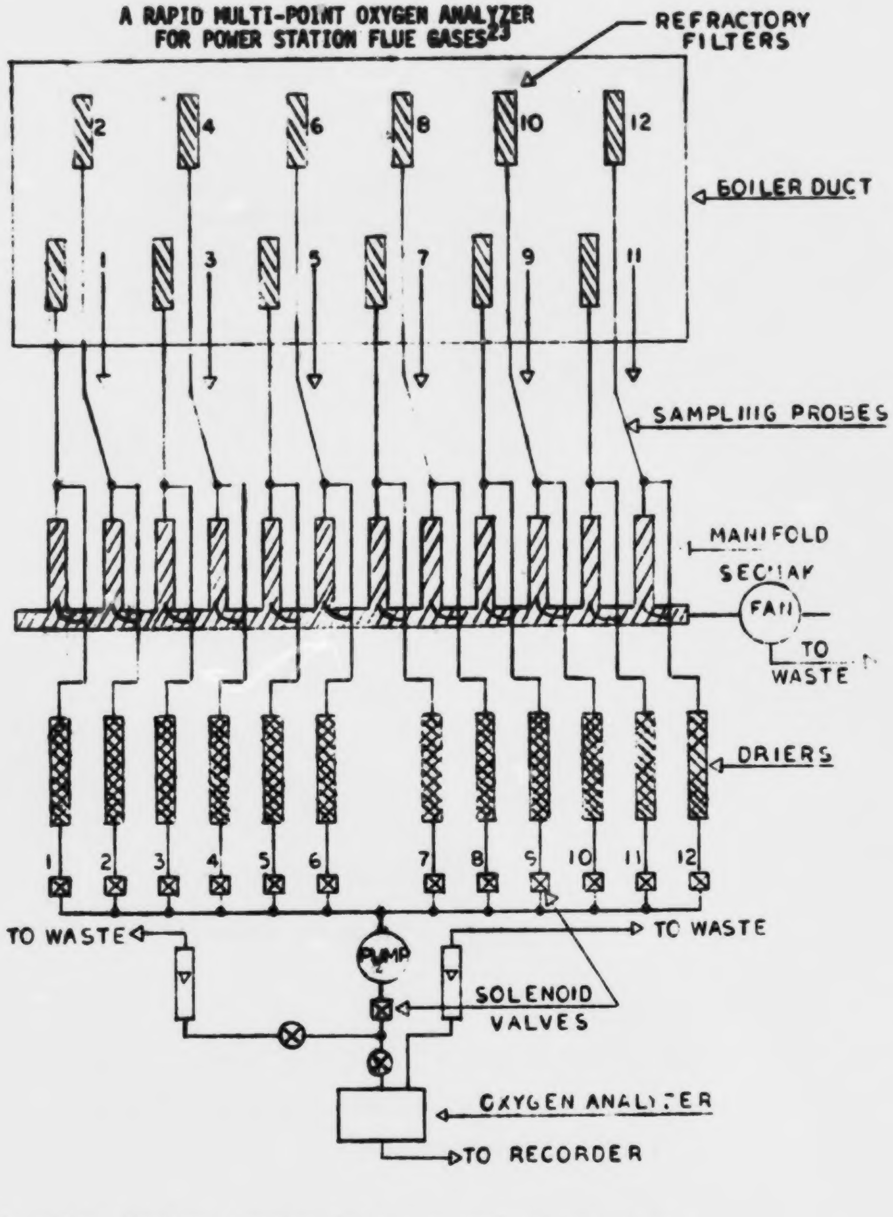
APPENDIX B

ARRAYS AND MECHANICAL TRAVERSING SYSTEMS

This appendix presents brief descriptions of several examples of sampling arrays and two mechanical probe traversing schemes.

FIGURE B 1

A RAPID MULTI-POINT OXYGEN ANALYZER FOR POWER STATION FLUE GASES[23]


REFRACTORY FILTERS
2
4
6
8
10
12
BOILER DUCT
1
3
5
7
9
11
SAMPLING PROBES
MANIFOLD SECTION
FAN
TO WASTE
DRIERS
1
2
3
4
5
6
7
8
9
10
11
12
TO WASTE
PUMP
TO WASTE
SOLENOID VALVES
OXYGEN ANALYZER
TO RECORDER

This system displays a twelve point scan that can carry out a traverse at the rate of one scan per minute. In this sequential operation the spatial variation of oxygen concentration in the section of the duct is obtained.

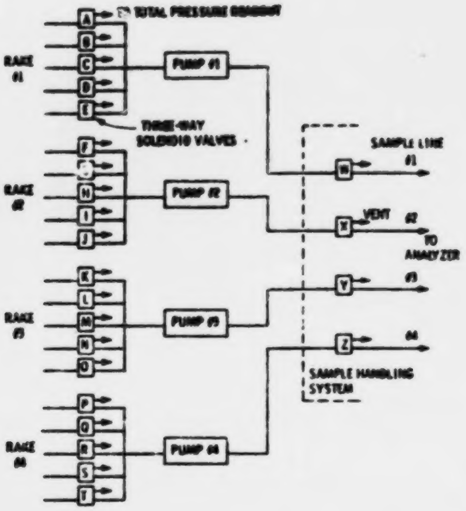

TOTAL PRESSURE READOUT
RAKE #1
RAKE #2
RAKE #3
RAKE #4
A
B
C
D
E
F
H
I
J
K
L
M
N
O
P
Q
R
S
T
PUMP #1
PUMP #2
PUMP #3
PUMP #4
THREE-WAY SOLENOID VALVES
SAMPLE LINE #1
W
X
VENT
#2
TO ANALYZER
Y
#3
Z
#4
SAMPLE HANDLING SYSTEM

FIGURE B 2

SAMPLING SYSTEM USED

FOR ON-LINE GAS ANALYSIS

OF JET ENGINE EXHAUST [24]

This system displays a sampling probe in the form of multi-element rakes. Sampling probes are also used for total pressure measurement.

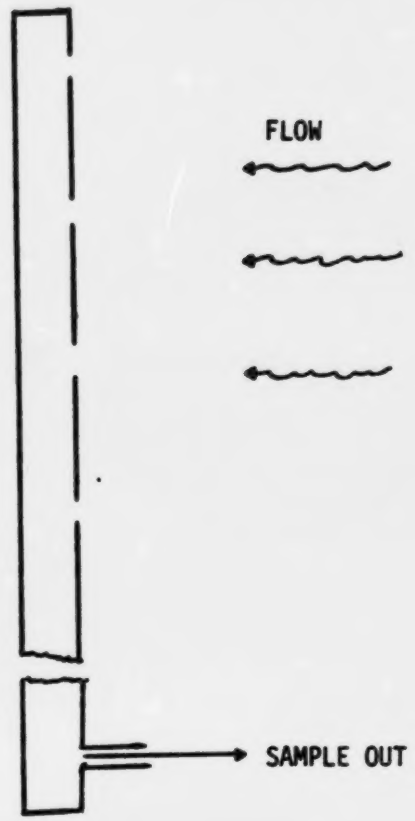


FIGURE B-3

PROBE WITH MULTIPLE SAMPLING HOLES

Representative sample is obtained when the flow is uniform. Design considerations regarding frictional pressure drop along the probe and non-equal quantities sampling in stratified concentrations are described in detail in Ref. 51. Probe length to diameter ratio must be less than 100 and the total sampling hole area less than 1/4 of the probe internal cross sectional area to obtain a good approximation of the spatial average of gas concentration.

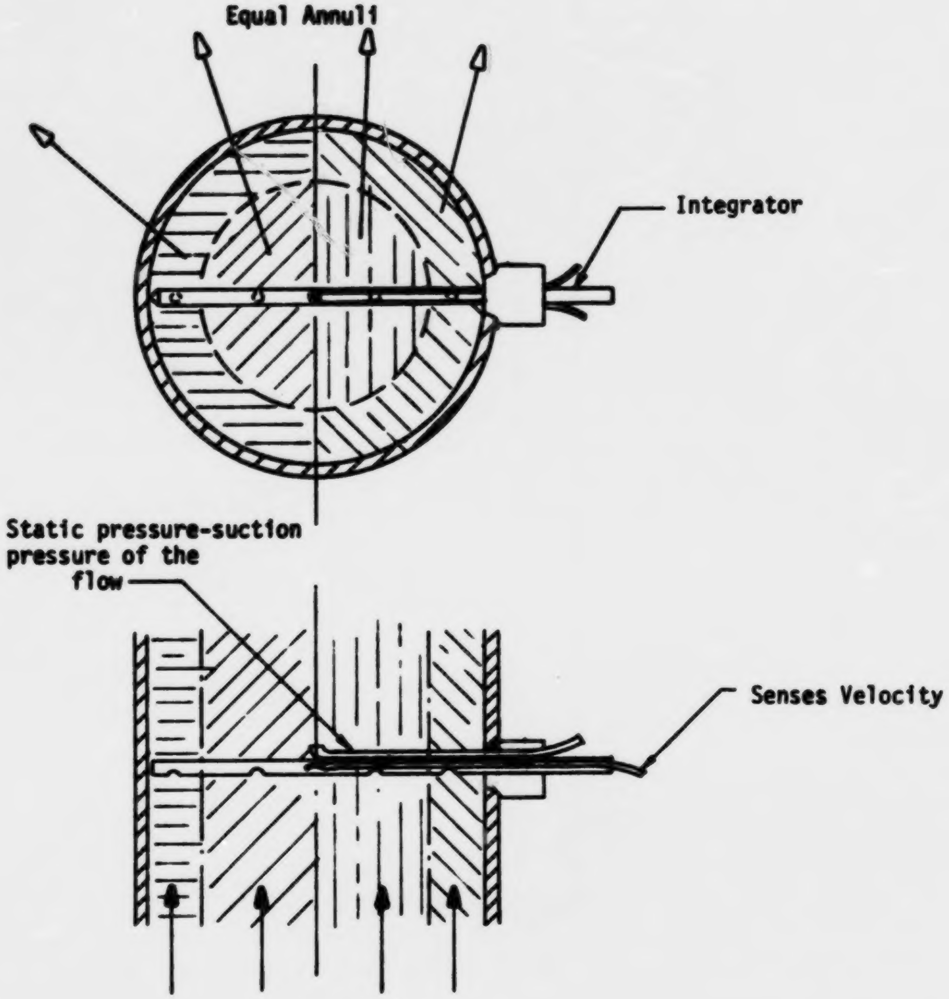

Equal Annuli
Integrator
Static pressure-suction pressure of the flow
Senses Velocity

FIGURE B-4
ANNUBAR FROM ELLISON INSTRUMENT DIV.[2]

This system displays an interpolating tube and sensing ports precisely located at predetermined positions. It uses the Chebyshef Calculus for averaging. The result of the measurement is an average velocity.

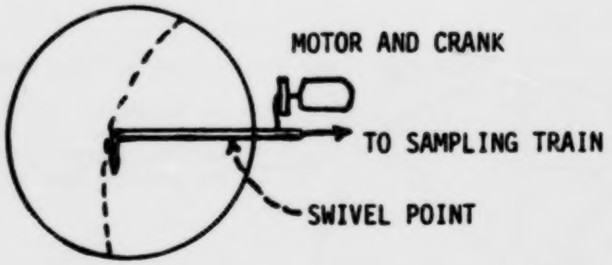

MOTOR AND CRANK
TO SAMPLING TRAIN
SWIVEL POINT

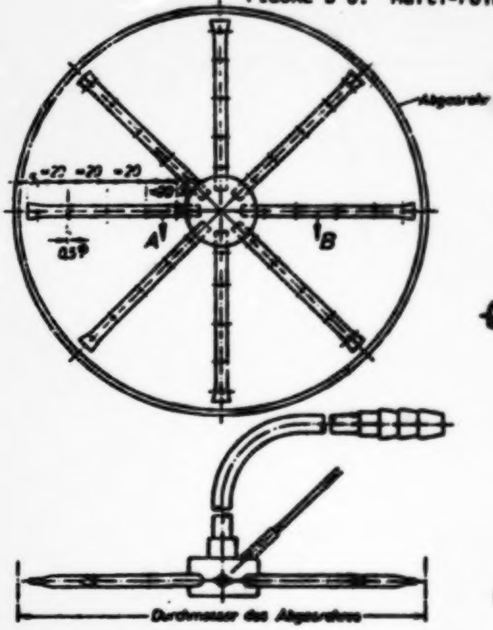
FIGURE B-5

CONSTANTLY-TRAVERSING SAMPLING PROBE[25,26]

A single sampling probe mechanically traverses in an arc over the flow channel while removing samples.

FIGURE B 6. Multi-Points Sampling Techniques in West Germany[49]

In rectangular ducts averaging is done along the diagonal axis of the cross section, including the corner points, and sampling through a common manifold.


Abgasrohr
≈20
≈20
≈20
A
B
0,5∅
Durchmesser des Abgasrohres

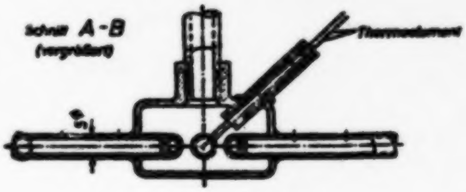

Schnitt A-B
(vergrößert)
Thermoelement

Bild 7 a. Absaugekreuz bei aufgesetzter Strömungssicherung mit Kreisquerschnitt

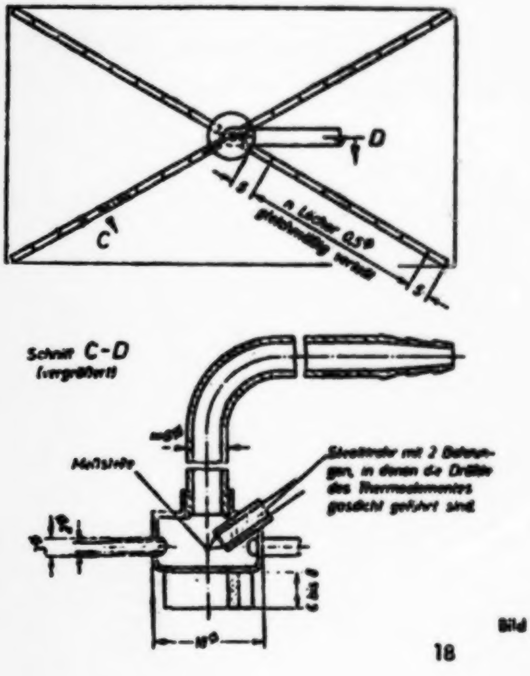

D
C
s
n Löcher 0,5∅
gleichmäßig verteilt
s
Schnitt C-D
(vergrößert)
Meßstelle
Stahlrohr mit 2 Bohrungen, in denen die Drähte des Thermoelementes gasdicht geführt sind

Bild 7 b. Absaugekreuz bei eingebauter Strömungssicherung, dem Abgaskanalquerschnitt angepaßt

APPENDIX C

DIFFUSION TUBE SAMPLING

C-1. IN STACK DIFFUSION TUBE

A special instrumental approach, the diffusion tube, has the potential to average concentration over part of the sample plane and control sample rate proportionally. A discussion on diffusion tubes is presented below.

Diffusion tubes are devices in which a sample of the species to be measured is extracted by permeation or diffusion through a surface without physical or bulk flow of the stream. The simplest configuration of a diffusion tube is a cylinder with its axis normal to the direction of flow in the duct. A carrier gas which mixes with and transports the sampled species to the monitor flows through the tube.

The rate at which material diffuses depends on the combined resistance to mass transfer through the stagnant boundary layer outside the tube, through the tube itself, and through the boundary layer inside. The concentration profile in the general case is shown in C-1 where C_∞ is the concentration in the bulk (the concentration we want to determine); C_o the concentration at the outer surface; and C_i the concentration at the inner surface. We assume the bulk concentration in the carrier stream is zero. In this case, assuming steady state flow, the rate of transport per unit area, n_a, is given by:

$$n_a = h_1 (C_\infty - C_o) = \frac{D}{t} (C_o - C_i) = h_2 C_i$$

where

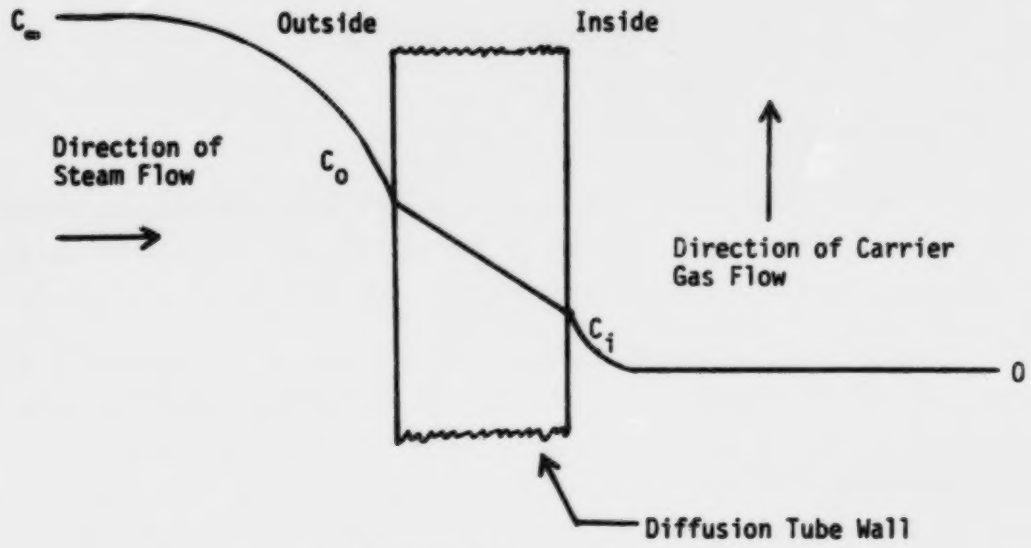
n_a = mass transport rate (gm/cm^2-sec)

h_1, h_2 = mass transfer coefficients on outer and inner sides, respectively (cm/sec)

C_∞, C_o, C_i = concentration (gm/cc)

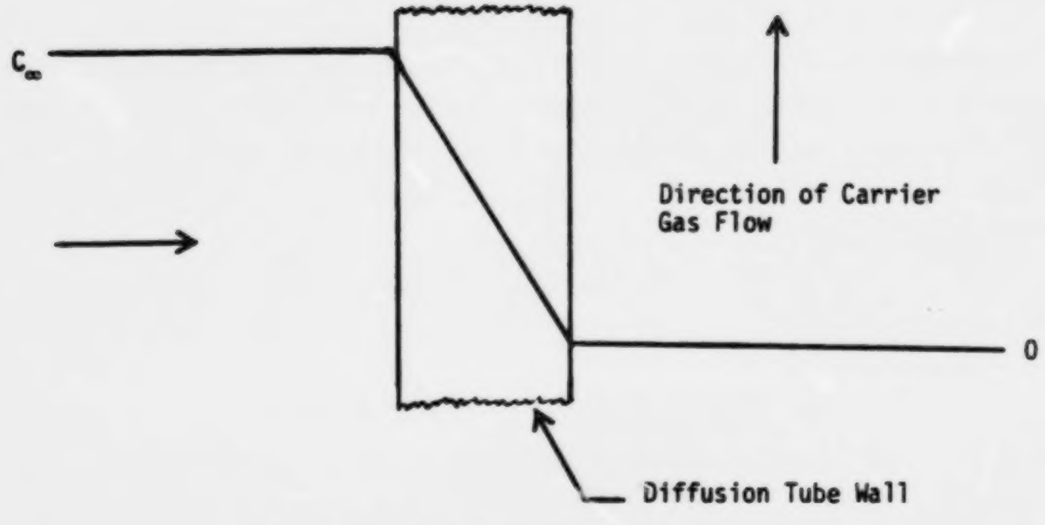
D = permeability of membrane (cm^2/sec)

t = thickness of membrane (cm)



C_∞
Outside
Inside
Direction of Steam Flow
C_o
Direction of Carrier Gas Flow
C_i
0
Diffusion Tube Wall

a. General Case



C_∞
Direction of Carrier Gas Flow
0
Diffusion Tube Wall

b. Permeation Rate Limited

Figure C-1. Concentration Profiles

These equations can be combined into a single relationship:

$$n_a = u\,C_\infty$$

and

$$\frac{1}{u} = \frac{1}{h_1} + \frac{1}{h_2} + \frac{t}{D}$$

where u = overall mass transfer coefficient.

Now if, as usually is the case, the rate of diffusion through the membrane is much lower than in the films, i.e.:

$$h_1 >> \frac{D}{t}$$

and

$$h_2 >> \frac{D}{t}$$

then the overall equation reduces to the simple form:

$$n = \frac{D}{t}\,C_\infty$$

If we now integrate the flow through a finite length (L) of tube of diameter (d), the total amount of material entering is:

$$N = \frac{D}{t}(\pi d)\int_L C_\infty\,dL$$

Further, the apparent concentration (S) as determined at the monitor is:

$$S = \frac{N}{q} = K\int_L C_\infty\,dL$$

where

S = measured concentration in gm/cc

q = flow rate of carrier gas, cc/sec

$K = \frac{D\pi d}{tq}$

Thus, if permeability controls, the sensed concentration is proportional to the integral of concentration along the tube, but not to the integral of concentration times velocity. It is this latter quantity that is proportional to emission through the duct. The diffusion tube then is not theoretically suited to applications in which emission rates are to be determined. On the other hand, it is an excellent device to obtain a sample proportional to the average value of concentration.

If this device is to be used in a circular duct, the diameter of the tube should increase along the radius proportional to the square of the radius, in order to obtain proper area weighting.

Materials such as teflon show finite permeation rates for SO_2, NO_2, and other small molecular species. The rates of diffusion through films of sufficient thickness to maintain mechanical integrity are so low that the rate controlling mechanism will be as assumed above. If the membrane permeability could be increased to the same magnitude as the outer boundary layer coefficient, then the flow along a section becomes:

$$N = \pi d \int_L u \, C_\infty \, dL$$

$$\frac{1}{u} = \frac{1}{h_1} + \frac{t}{D}$$

and

$$u = \frac{h_1 D}{D + t h_1}$$

Thus

$$N = \frac{D\pi d}{t} \int \frac{h_1}{D/t + h_1} \, D_\infty \, dL$$

the film coefficient, h_1, is a complex function of velocity, depending on the flow regimes, orientation of tube, etc.

C-2. CALCULATION FOR OUT-OF-STACK DIFFUSION TUBE

This appendix shows the detailed calculations for the out-of-stack diffusion tube. Figure C-1 shows a conceptual design for an out-of-stack diffusion tube and Figure C-2 shows the conditions for this calculation. Figure C-3 shows a conceptual design for a multi-element out-of-stack diffusion tubes.

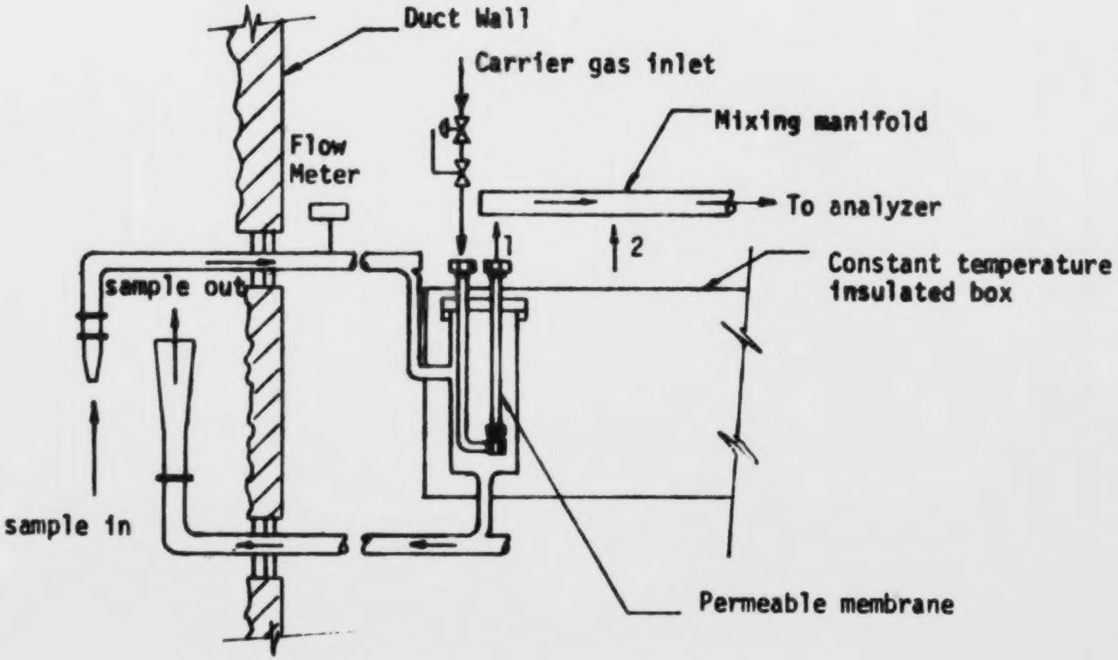
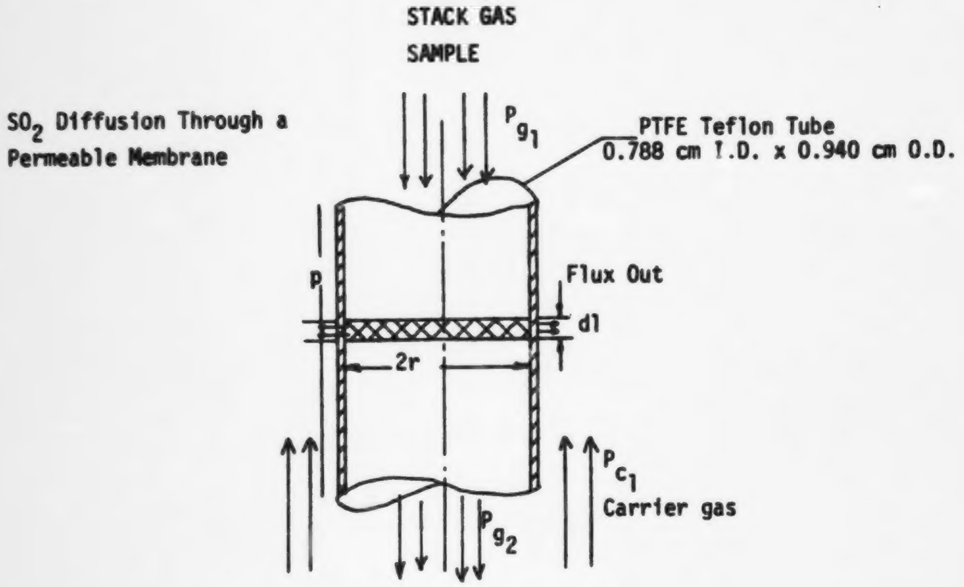

Duct Wall
Carrier gas inlet
Mixing manifold
Flow Meter
To analyzer
1
2
Constant temperature insulated box
sample out
sample in
Permeable membrane

FIGURE C-1
CONCEPTUAL DESIGN OF AN OUT-OF-STACK DIFFUSION TUBE
(FOR ONE ELEMENT SHOWN)


STACK GAS
SAMPLE
SO_2 Diffusion Through a
Permeable Membrane
P_{g_1}
PTFE Teflon Tube
0.788 cm I.D. x 0.940 cm O.D.
P
Flux Out
dl
2r
P_{c_1}
Carrier gas
P_{g_2}

Permeability of SO_2 for PTFE
PTFE Teflon Tube
0.788 cm I.D. x 0.94 cm O.D.

T°C	SO_2 permeability P cm Hg x cm^3 SO_2(STP)/cm^2-sec^{-1} - cm^{-3}(pol.)
121°	0.539×10^{-8}
177°	1.81×10^{-8}
232°	4.48×10^{-8}

FIGURE C-2

CONDITIONS

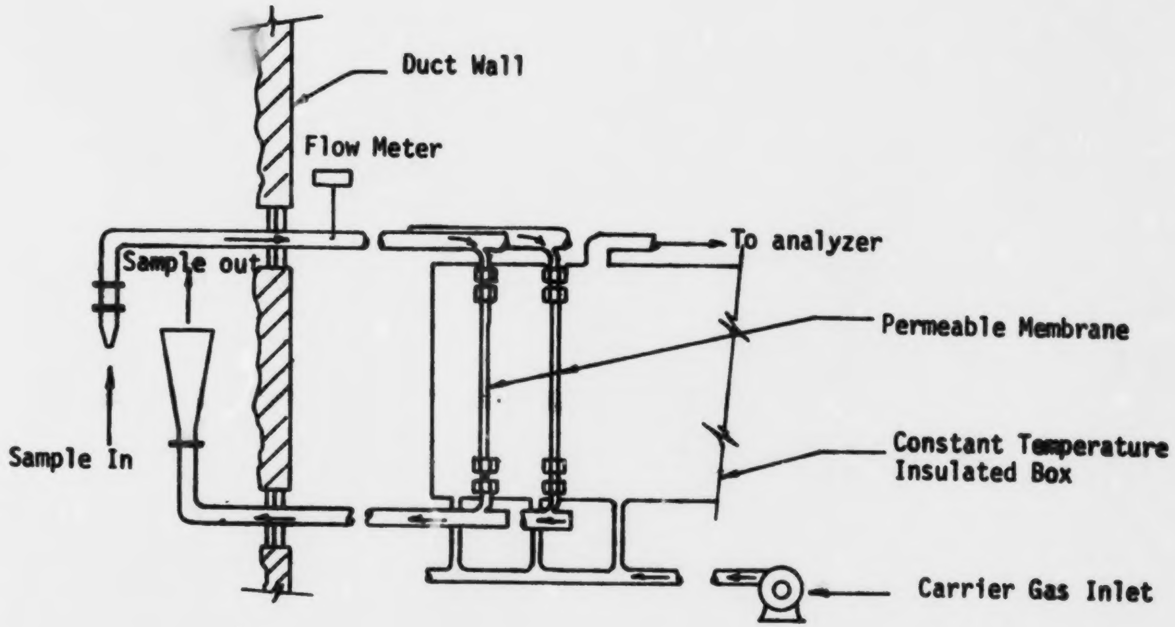

Duct Wall
Flow Meter
To analyzer
Sample out
Permeable Membrane
Sample In
Constant Temperature Insulated Box
Carrier Gas Inlet

FIGURE C-3

MULTI-ELEMENT OUT-OF-STACK DIFFUSION TUBES

The permeability is a function of temperature. The equation can be written as follows:

$$P = P_o \exp(-E_p/RT) \qquad (1)$$

To find P_o for the above tube we can write:

$$\frac{P}{P_o} = \exp(-E_p/RT)$$

$$\ln \frac{P}{P_o} = \frac{-E_p}{RT}$$

$$2.3 \left[\log P - \log P_o\right] = \frac{-E_p}{RT} \qquad (2)$$

At 121°C $P = 0.539 \times 10^{-8}$

Replacing the values in Equation (2)

$$2.3 \left[\log 0.539 \times 10^{-8} - \log P_o\right] = -\frac{6990}{1.987 \times 394}$$

$$\log 0.539 + \log 10^{-8} + \frac{6990}{1.987 \times 2.3 \times 394} = \log P_o$$

$$-0.2684 - 8 \times 1 + 3.882 = \log P_o$$

$$-4.3864 = \log P_o$$

$$P_o = 4.1 \times 10^{-5}$$

$$P = 4.1 \times 10^{-5}\ e^{-\frac{6990}{1.987\,T}}$$

$$P = 4.1 \times 10^{-5}\ e^{-\frac{3517.86}{T}} \qquad (3)$$

It is convenient to express the flux "F" in terms of the bulk-phase partial pressures of the permeating gas ' for steady state and diffusion of a gas from a hollow cylinder. Also assuming a constant diffusity and the applicability of Henry's Law:

$$F = \frac{2\pi P(pg-pc)}{\ln a/b} \tag{4}$$

where:

$$P = \frac{cm^3\ (STP) - cm^2}{sec\text{-}au^3\ (polymer)\text{-}cm\ Hg}$$

pg = partial pressure of SO_2 in gas sample, cm(Hg)
pc = partial pressure of SO_2 in the carrier gas, cm(Hg)
a,b = inner and outer tube radii, respectively, air
F = diffusive flux, cm^3 (STP)/sec - cm length

replacing P, equation (4) becomes

$$F = \frac{2\pi\left(4.1 \times 10^{-5} \quad \frac{-3517.86}{T}\right)(pg-pc)}{\ln a/b}$$

By assuming plug flow and neglecting all resistances except the tube membrane, we can write

$$\frac{dy}{de} = F' \qquad y = \text{mole fraction}$$

By neglecting pc, the partial pressure in carrier gas being very low it may be neglected, also if $pg = P_{tg} \times y$

$$F = \frac{2\pi\left(4.1 \times 10^{-5} \quad \frac{-3517.86}{T}\right) P_{tg}\ y_{SO_2}}{\ln a/b} \tag{5}$$

where

P_{tg} is the total pressure of the gas stream in cm(Hg)
y_{SO_2} is the mole fraction of SO_2 in the stream

If the ideal gas law holds for SO_2 $\quad PF = \eta' RT$ at STP

where η' = number of moles diffusing/sec - cm length

$$\eta' = \frac{F/10^3 \times 1}{0.082 \times 273} \tag{6}$$

also for the gas stream $\quad PQ = NRT$

where N = Total number of moles/unit time in gas stream
Q = Flow rate reduced to STP in cm^3/sec

$$N = \frac{Q/10^3 \times 1}{0.082 \times 273} \tag{7}$$

$$\frac{\eta'}{N} = \frac{F}{Q} \tag{8}$$

Also $\quad \frac{dN}{dl} = \eta' \quad$ but $\frac{dN}{N} = dy$ (assuming $dNy = \cancel{ydN}^{=0} + Ndy$)

$$Ndy = \eta' dl$$

$$\frac{dy}{dl} = \frac{\eta'}{N} = F' \tag{9}$$

by substitution

$$\frac{dy}{dl} = \frac{F}{Q} = \frac{2\pi\left(4.1 \times 10^{-5} e^{\frac{-3517.86}{T}}\right) P_{tg}{}^{SO}{}_2}{\ln a/b \quad \times \quad Q}$$

$$\int \frac{dy}{d} = \frac{2\pi\left(4.1 \times 10^{-5}\, e^{\frac{-3517.86}{T}}\right)P_{tg}}{\ln a/b \times Q}\int dl$$

$$\frac{\left(\ln y_2 - \ln y_1\right)\left(\ln a/b\right) \times Q}{2\pi\left(4.1 \times 10^{-5}\, e^{\frac{-3517.86}{T}}\right)P_{tg}} = l(\mathrm{cm}) \qquad (10$$

if $T = 100°C = 373°K$

$Y_2 = 50\ \mathrm{ppm} = 50/10^6$

$Y_1 = 5000\ \mathrm{ppm} = 5000/10^6$

$a = 0.788$

$b = 0.940$

$P_{tg} = 76\ \mathrm{cm\ Hg}$

$Q = 3\ \mathrm{liters/min} \equiv 50\ \mathrm{cm}^3/\mathrm{sec}$

$$\frac{\left(\ln 50 \times 10^{-6} - \ln 5000 \times 10^{-6}\right)\left(\ln \frac{0.788}{0.940}\right) \times 50}{2\pi\left(4.1 \times 10^{-5}\, e^{\frac{-3517.86}{373}}\right) \times 76} = l(\mathrm{cm})$$

$$l = 0.2612 \times 10^{8}\ \mathrm{cm}$$

APPENDIX D

TEMPERATURE STABILIZED DIFFUSION TUBE

A proposed method to stabilize the temperature of a diffusion tube was brought to our attention in a private communication with J.J. McKinley (president of KIN-TEK Laboratories). In this scheme, a diffusion tube is wrapped as a helix around a heat pipe (Figure D-1) which is a temperature adjustable, isothermal tube. The close contact between the diffusion tube and the heat pipe maintains the diffusion at a constant temperature.

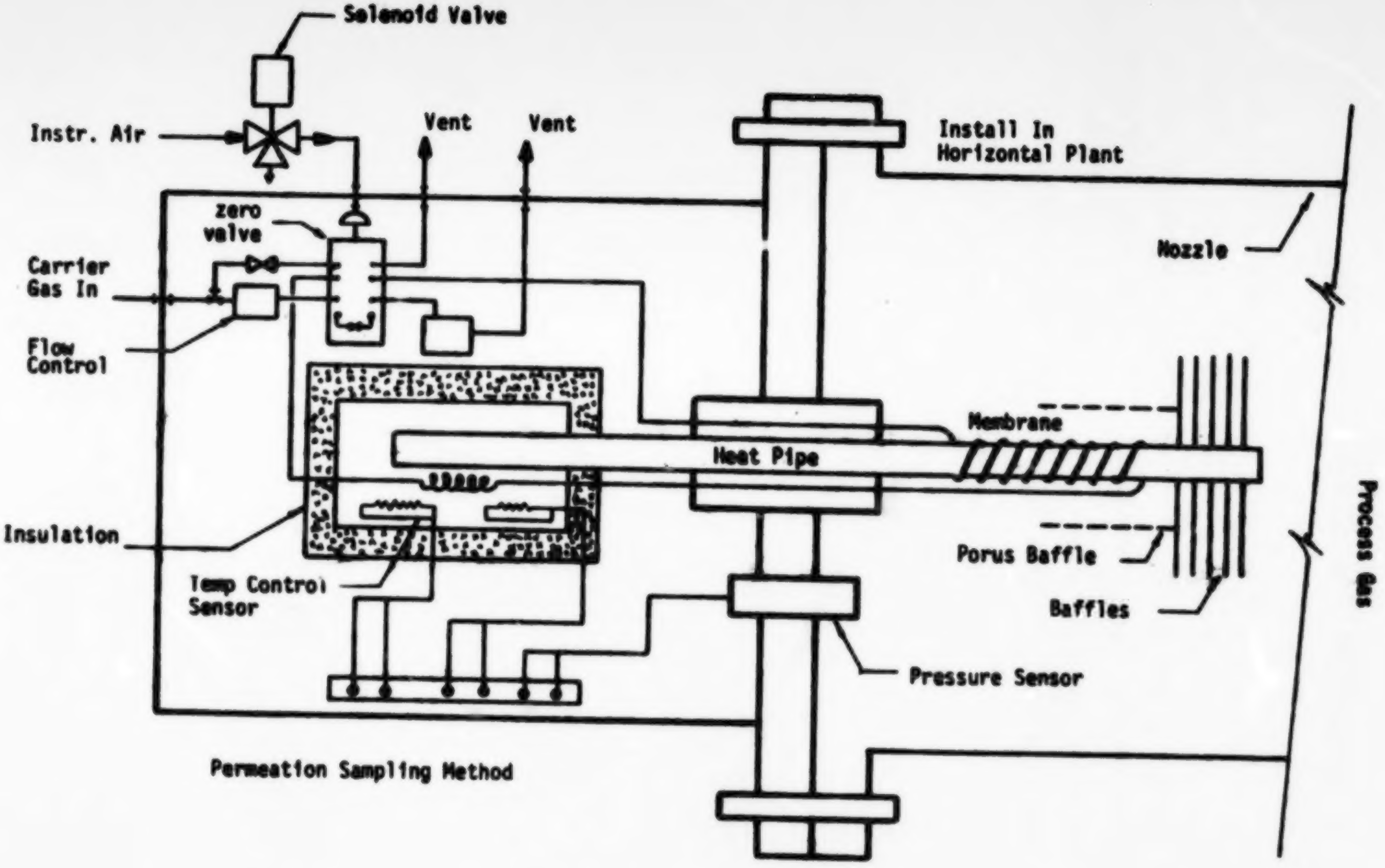

Solenoid Valve
Instr. Air
Vent
Vent
Install In
Horizontal Plant
zero valve
Carrier Gas In
Flow Control
Nozzle
Membrane
Heat Pipe
Process Gas
Insulation
Porus Baffle
Temp Control Sensor
Baffles
Pressure Sensor
Permeation Sampling Method

FIGURE D-1

SCHEMATIC OF TYPICAL IN-LINE SAMPLER

APPENDIX E

TRACER METHODS

In theory, it is implicitly possible to weight, an average value of concentration obtained by uniform (non-proportional) sampling to reflect the proper velocity-concentration product average needed to provide a measure of emissions. This can be done by reference to a tracer compound introduced into the flowing stream externally, or released or produced during the same processes forming the material to be determined. If the emission rate of the tracer is known by reference to the overall stoichiometry of the process or by metering, then the emission rate of the desired species can be determined. The basis of this technique is developed in the following analysis.

Assume we are using a multi-point sampling array, with N identical probes, each of area a, and with a sampling velocity of v_s. Assume also that each is located within an equal area (A) segment of the duct. (It is not necessary to make this latter assumption, but it is less complex analytically.)

The amount of tracer and species sampled per unit time are given by:

$$e_{t \text{ or } x} = \sum_{N} v_s a \, C_{t,i \text{ or } x,i} = a v_s \sum_{N} C_{t,i \text{ or } x,i}$$

where v_s = sample probe inlet velocity

a = sample probe area

$C_{t,i}, C_{x,i}$ = concentration of tracer and species, respectively, at i^{th} probe

e_t, e_x = rate of sampling of tracer and species, respectively

The emission rate of the two materials through the system is given by:

$$m_t = A \sum v_i \, C_{t,i}$$

$$m_x = A \sum v_i \, C_{x,i}$$

where

A = area associated with each probe

v_i = actual stream velocity at i^{th} probe

m_t, m_x = emission rate of tracer and species, respectively

Now take the ratios of sampled rate to emission rate:

$$\frac{e_t}{m_t} = \frac{a\, v_s}{A} \frac{\sum C_{t,i}}{\sum v_i\, C_{t,i}}$$

Now, if the concentration profile of the tracer is proportional to the profile of the contaminant species, i.e.,

$$C_{t,i} = k\, C_{x,i}$$

then the two ratios are equal.

$$\frac{e_t}{m_t} = \frac{e_x}{m_x}$$

or

$$m_x = m_t \frac{e_x}{e_t} = m_t \frac{\sum C_{x,i}}{\sum C_{t,i}} = m_t \frac{S_x}{S_t}$$

where S_x, S_t are the sensed signals from the monitor in idential units.

Thus, if we know the emission rate of the tracer, the emission rate of the unknown species can be determined. In combustion systems, one tracer of particular interest appears to be CO_2. This specie is produced at the same time and in the same space as the other gaseous pollutants, SO_2, NO_x, etc., so that it should exhibit an identical concentration profile. Further, the emission rate can be determined accurately by reference to the fuel rate and fuel ultimate analysis. This method, in theory, is valid for a single sampling point.

APPENDIX F

THERMAL NULL PROBES

Null probes, which use thermocouples to detect an error signal between the stack gas velocity and the sample gas velocity, potentially offer instrumental simplicity. The following analysis[27] considers the magnitude of the error signal produced due to differences in stagnation temperatures between the stack gas stream and the sample gas stream.

The total (stagnation) temperature (T_t) is equal to the sum of the static temperature (T_s) and the dynamic temperature of the gas (T_v). For an ideal probe at rest with respect to the system boundaries, the stagnation of an idealized gas follows the energy equation which may be written for the open system, viz.:

$$\delta Q + \delta W = dh + \frac{VdV}{g} \qquad (1)$$

δQ = heat transferred across system boundaries

δW = mechanical work transferred across system boundaries

dh = $C_p\, dT_s$ = enthalpy change between two thermodynamic states within the system.

In the absence of heat transfer (adiabatic) and mechanical work, the equation (1) may be integrated as follows:

$$Mg\, J\, C_p\, (T_2 - T_1) = 1/2M\, (V_1^2 - V_2^2) \qquad (2)$$

where J = 778 ft-lb/BTU

and

$$T_t = T_s + \frac{V^2}{2JgC_p} = T_s + T_v \qquad (3)$$

Since in real systems both heat transfer and friction are not absent, a correction factor is introduced. The temperature indicated by a real probe immersed in a real gas may be expressed as:

$$T_t - T_p = (1-K)\, T_v \qquad (4)$$

or $$T_p = T_s + K\,T_v$$

Where T_p is the indicated probe temperature, the value of K is a function of probe geometry. For the half-shielded type thermocouple, K is about 0.96.

In the thermal null method we are only interested in differences in temperature between the flue gas and the sampled gases. If the two thermocouples are close enough, T_s will be the same, but the value of T_v will change if the velocity of the flue gases does not equal the velocity of the gases at the entrance of the sampling nozzle. For example:

If $V_f = 19$ ft/sec

$V_s = 20$ ft/sec

and $C_p = 0.24$ BTU/lb°F

$$T_{v_f} = \frac{(19)^2}{2 \times 778 \times 32.2 \times 0.24} = 0.030°F$$

$$T_{v_s} = \frac{(20)^2}{2 \times 778 \times 32.2 \times 0.24} = 0.033°F$$

If K is taken as 0.96, the difference becomes:

$$0.96 \left[T_{v_s} - T_{v_f} \right] = 0.003°F$$

This temperature difference is very small. If a double junction, iron-constantan thermocouple is used for a differential temperature sensor, an EMF of about 0.9 μv must be detected. The detection of this magnitude of voltage is within the state of the art of present electronic technology. However, some development effort would be required for a system suitable for industrial conditions.

APPENDIX G

VELOCITY AND CONCENTRATION PROFILES

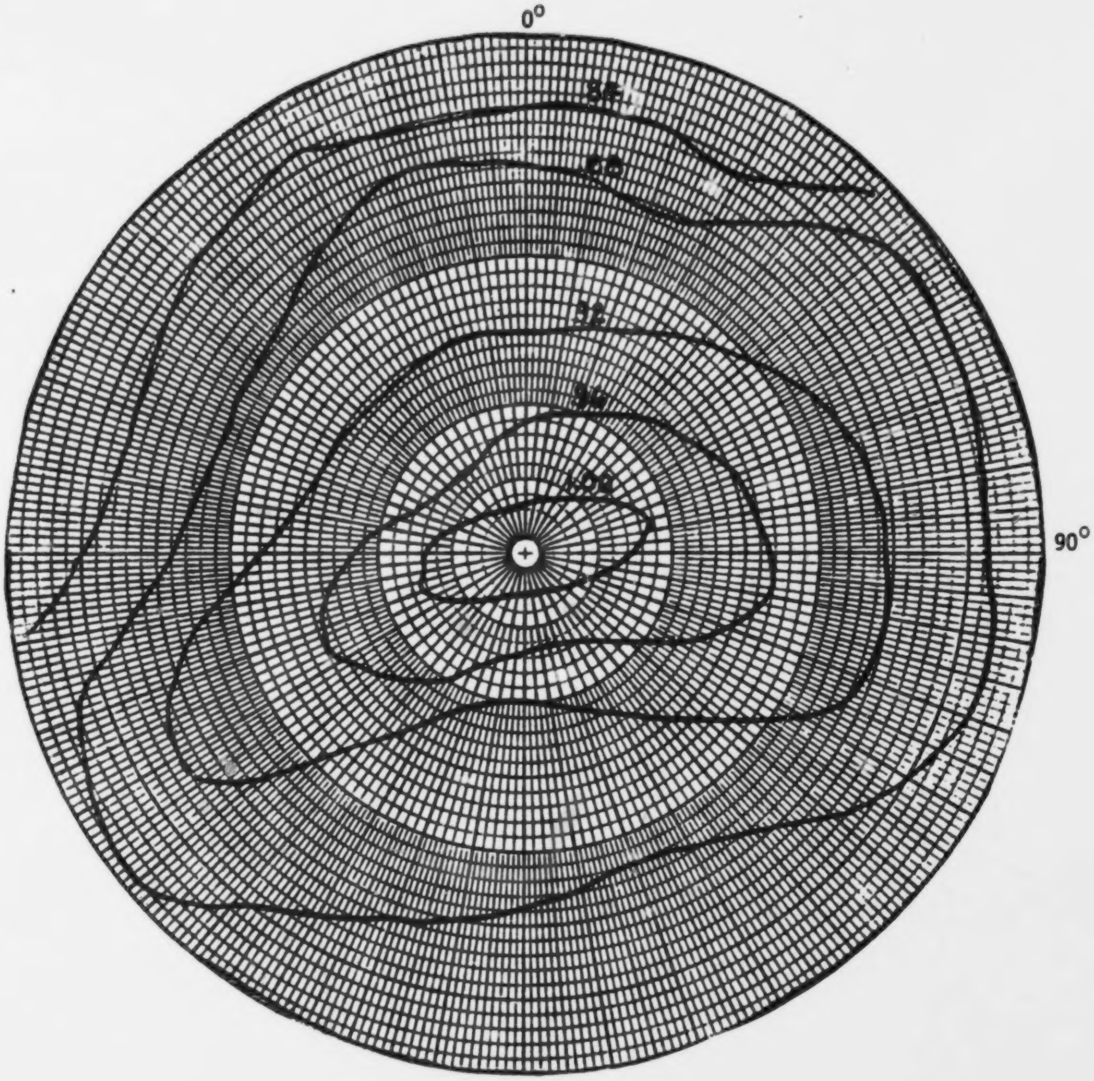


Figure 9-a. Case III Normalized Velocity Profile
8.0' Diameter Duct

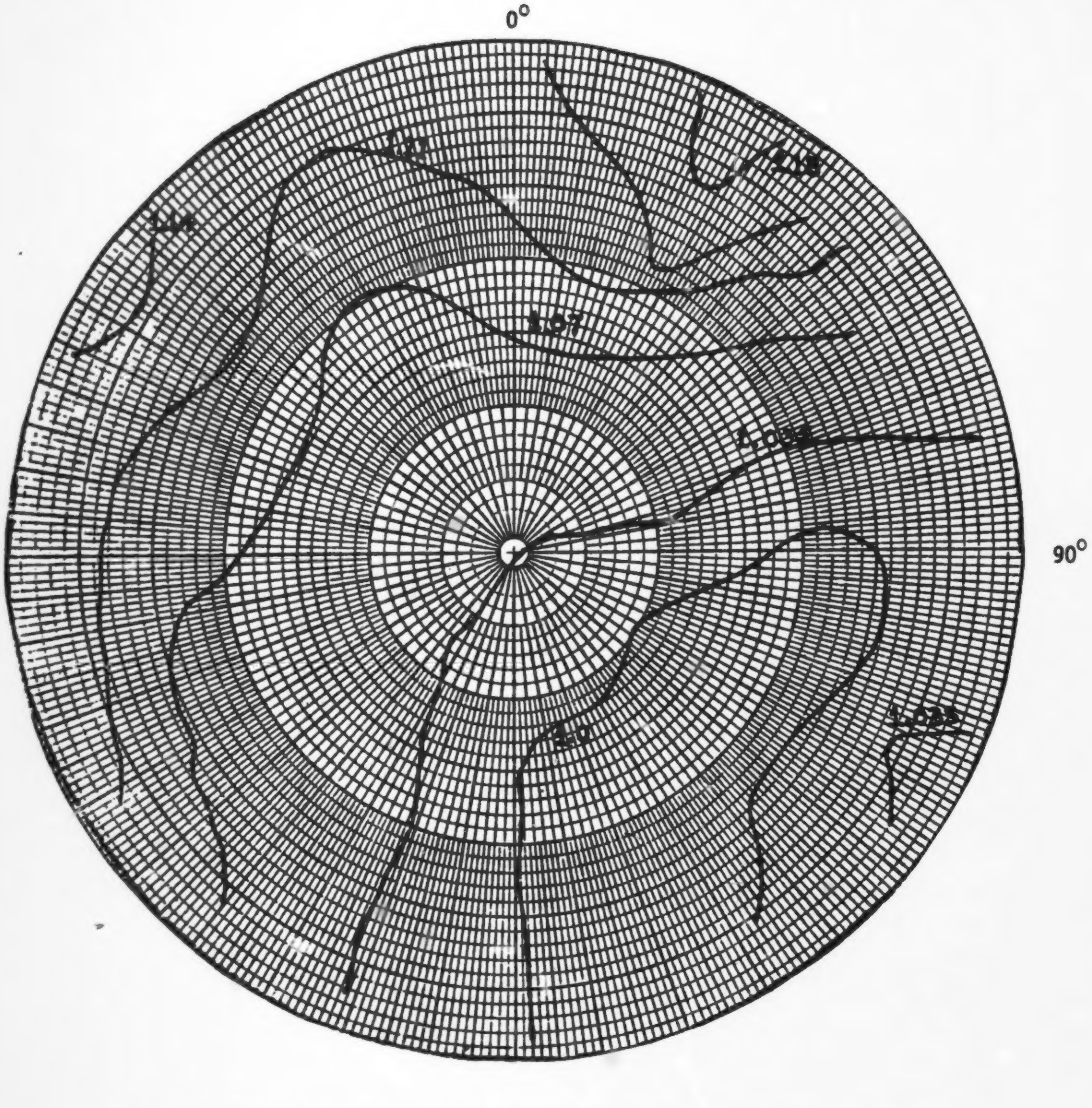


Figure 9-b. Case III Normalized Concentration Profile
8.0' Diameter Duct

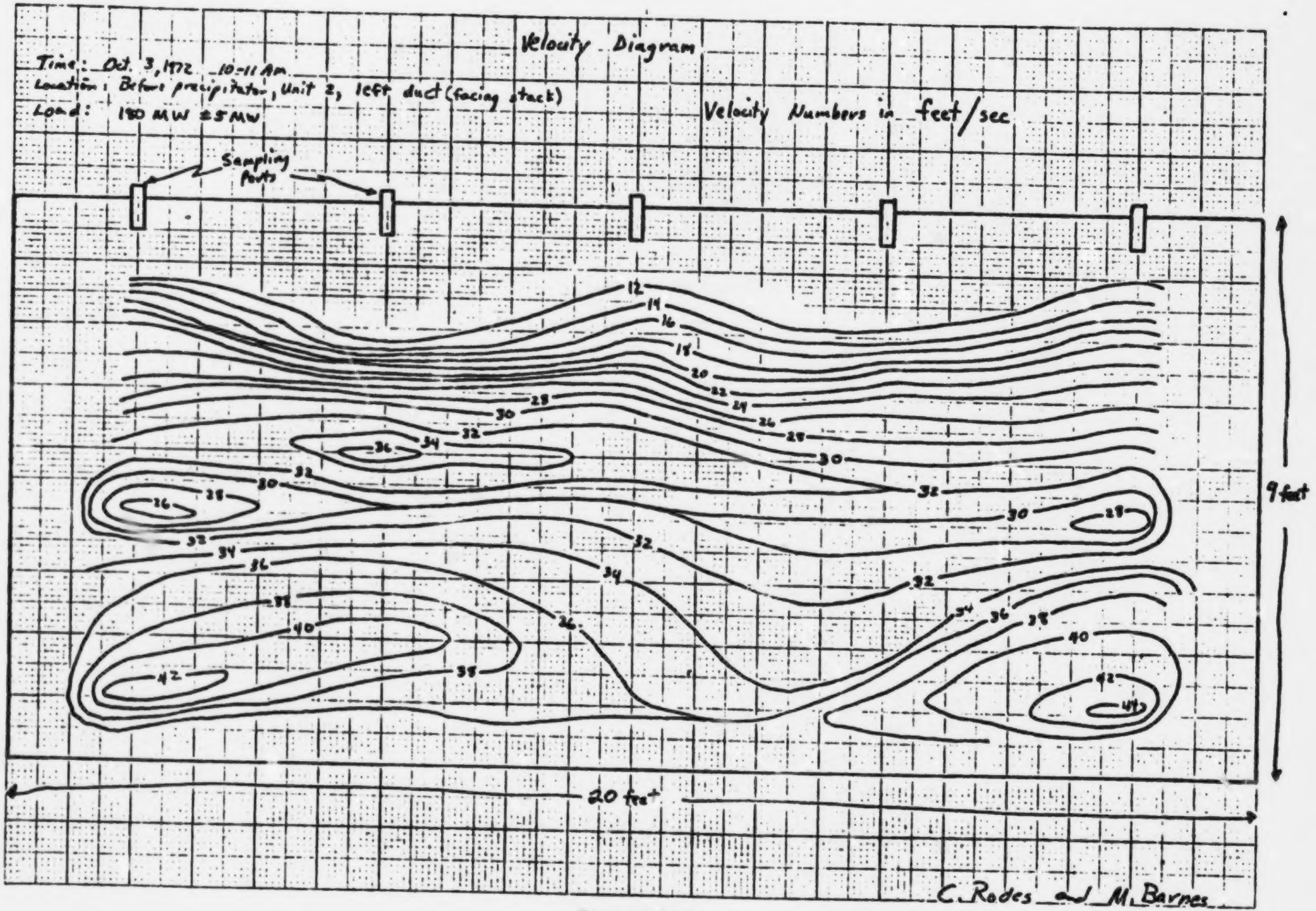


Figure 10-a.

VELOCITY PROFILE CASE IV

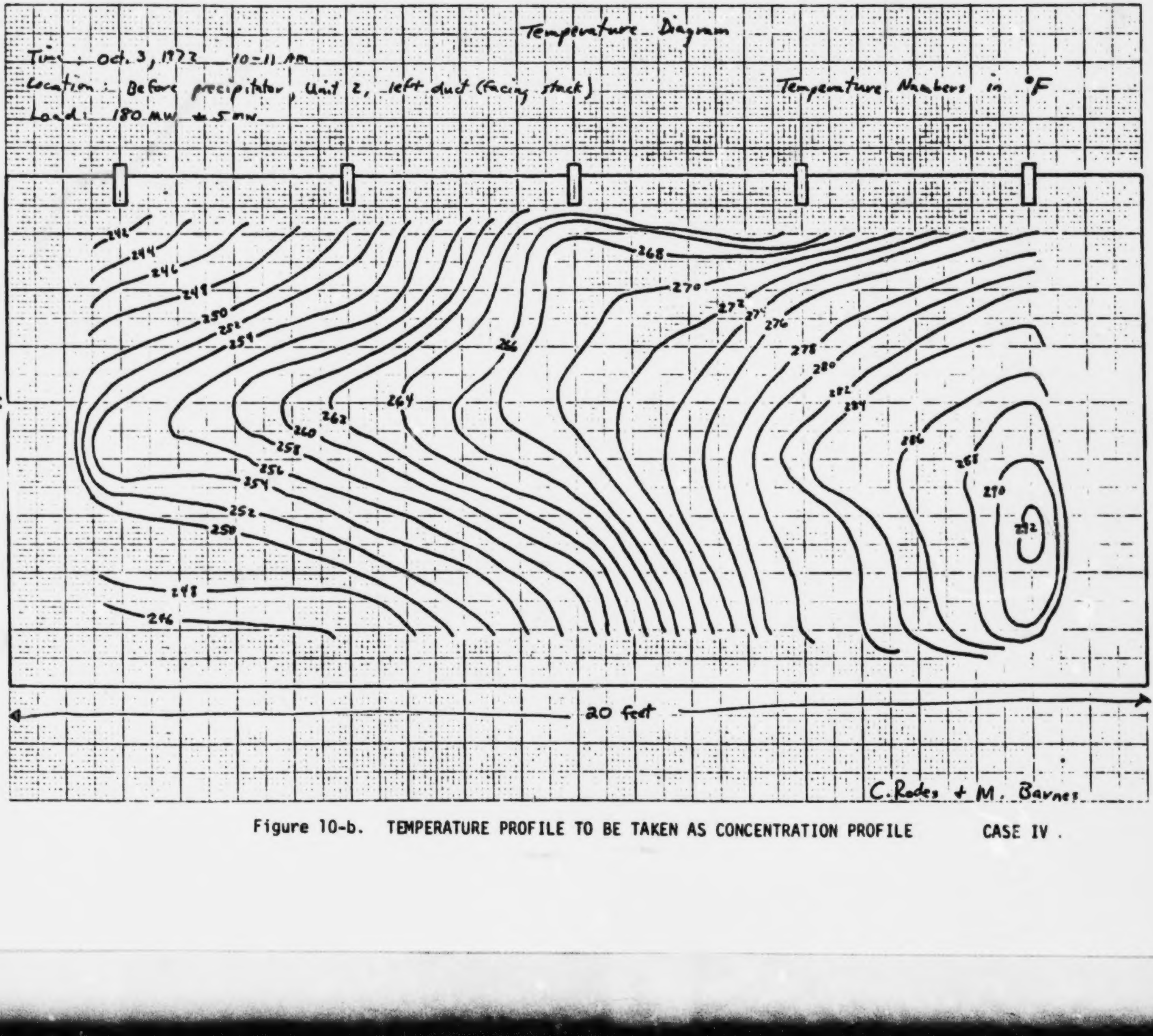
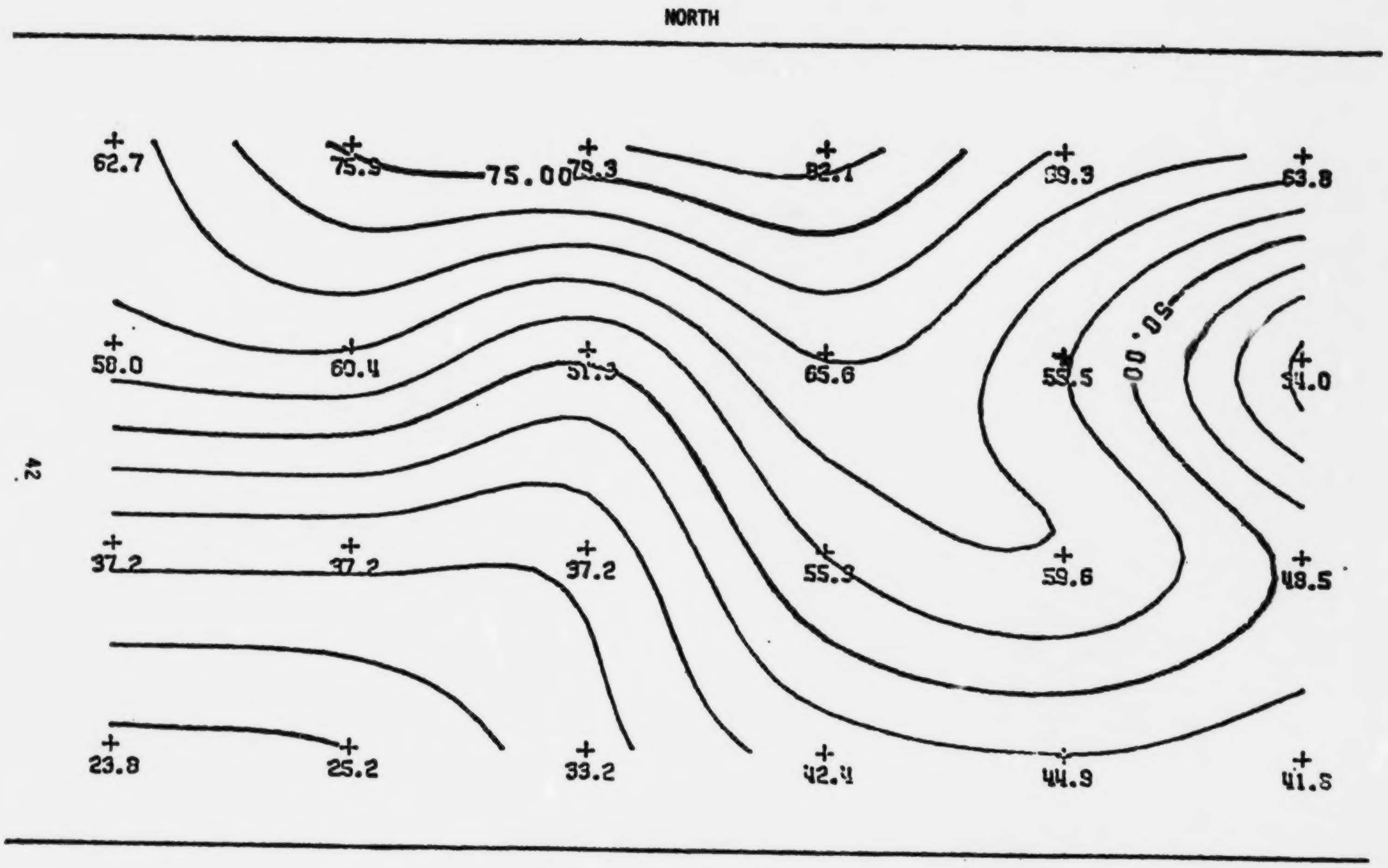


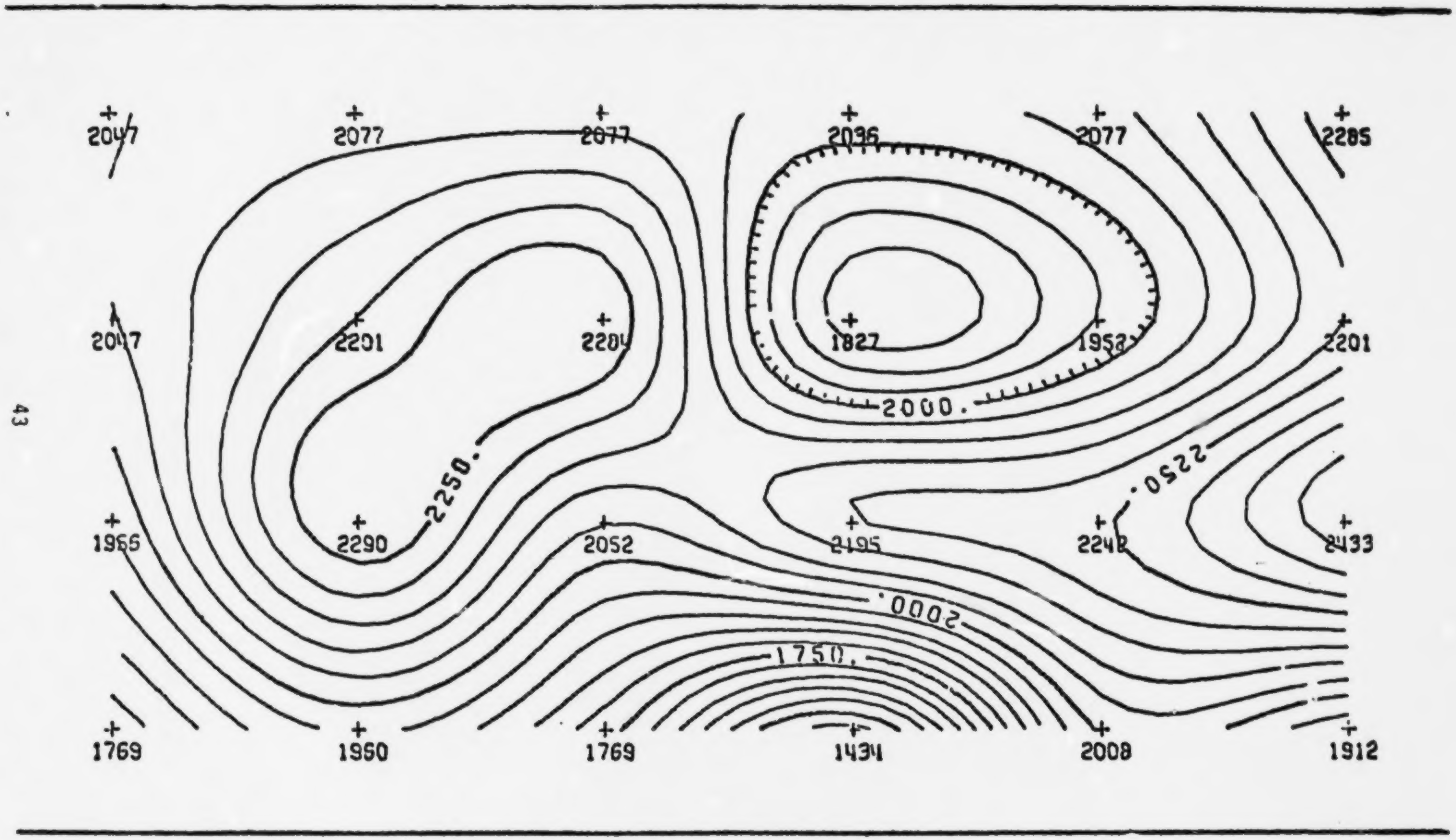
Figure 10-b. TEMPERATURE PROFILE TO BE TAKEN AS CONCENTRATION PROFILE CASE IV.



CASE V Figure 11a ([illegible])

ISOVELS - INTERVAL = 5 FT/SEC

PLANE A-A WEST SIDE 3-16-70



CASE V Figure 11b (TVA) 46 feet x 24 feet

S02 DISTRIBUTION - INT =50 PPM

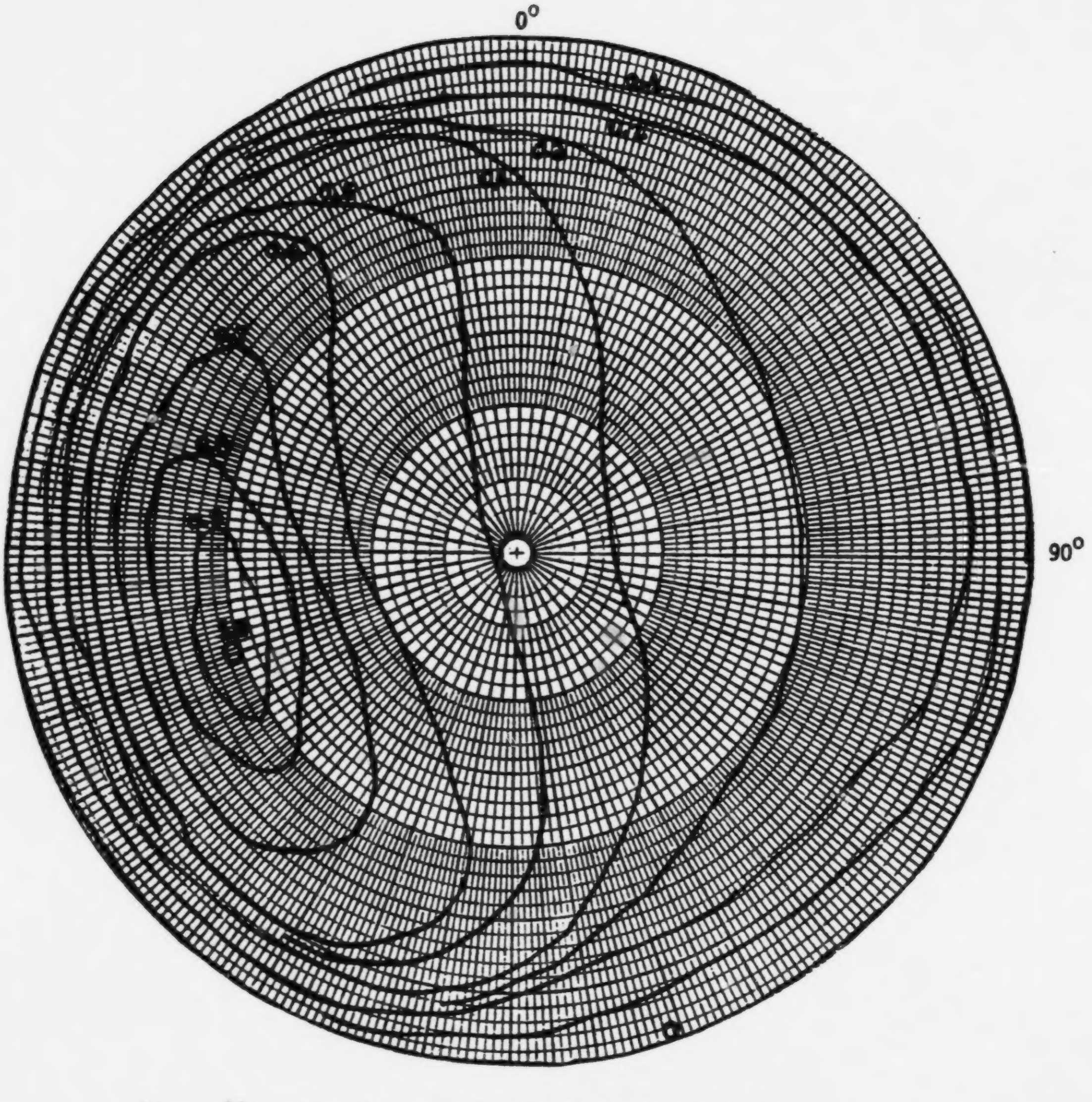


Figure 12a Case VI Hypothetical Velocity Profile
20' Diameter Duct

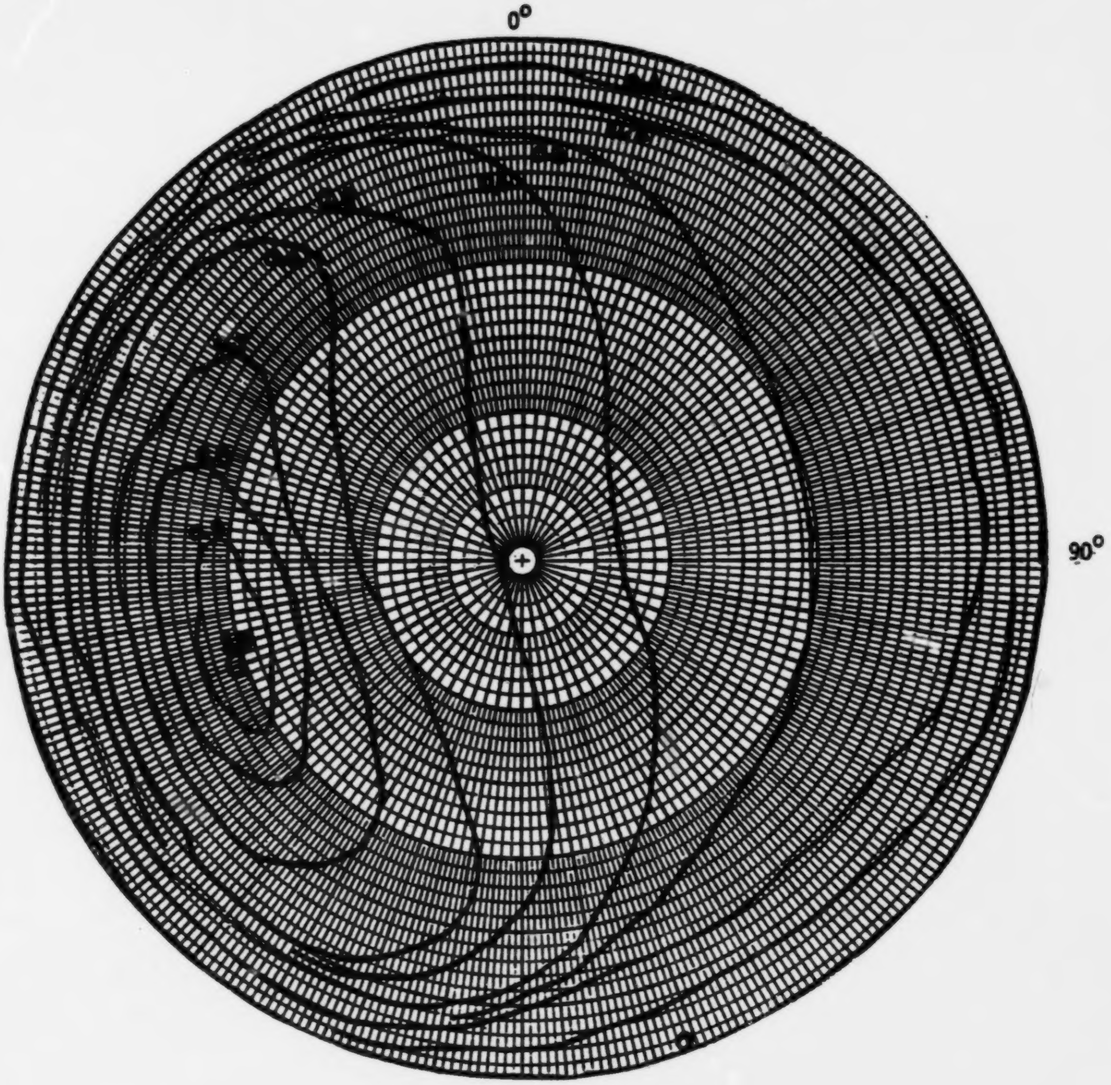


Figure 12b. Case VI Hypothetical Concentration Profile
20' Diameter Duct

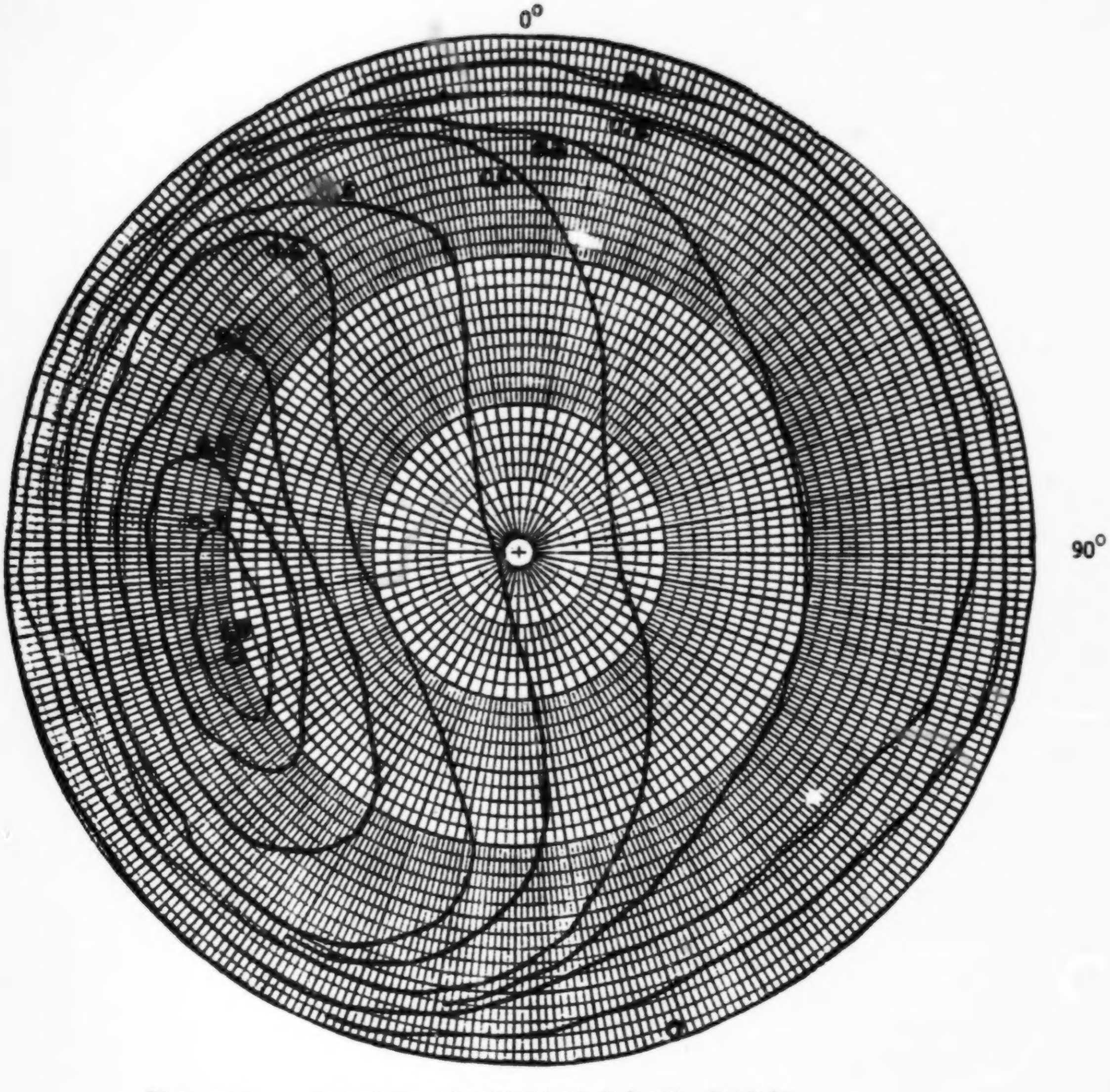


Figure 13a. Case VII Hypothetical Velocity Profile
20' Diameter Duct

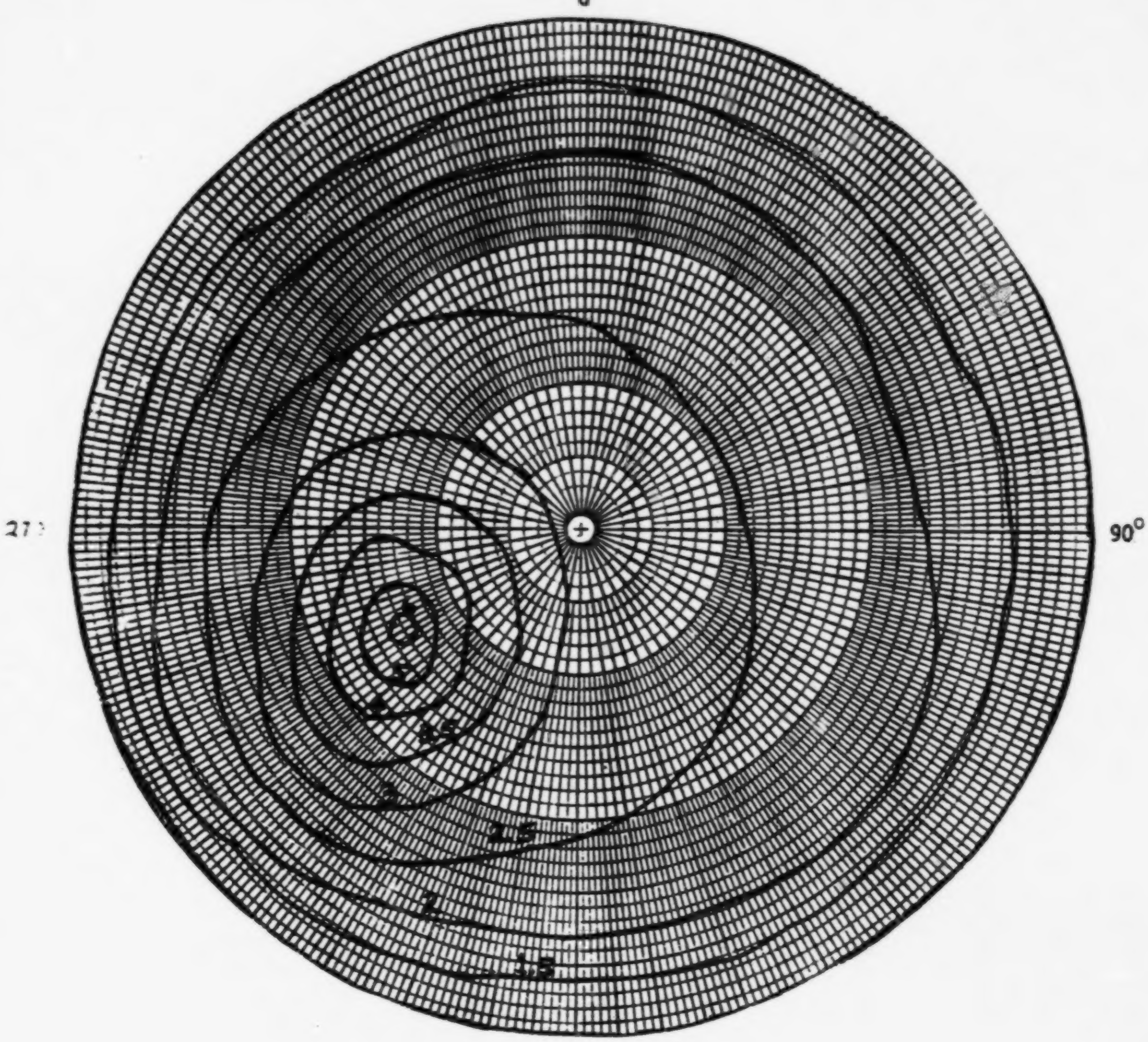


Figure 13b. Case VII Hypothetical Concentration Profile
20' Diameter Duct

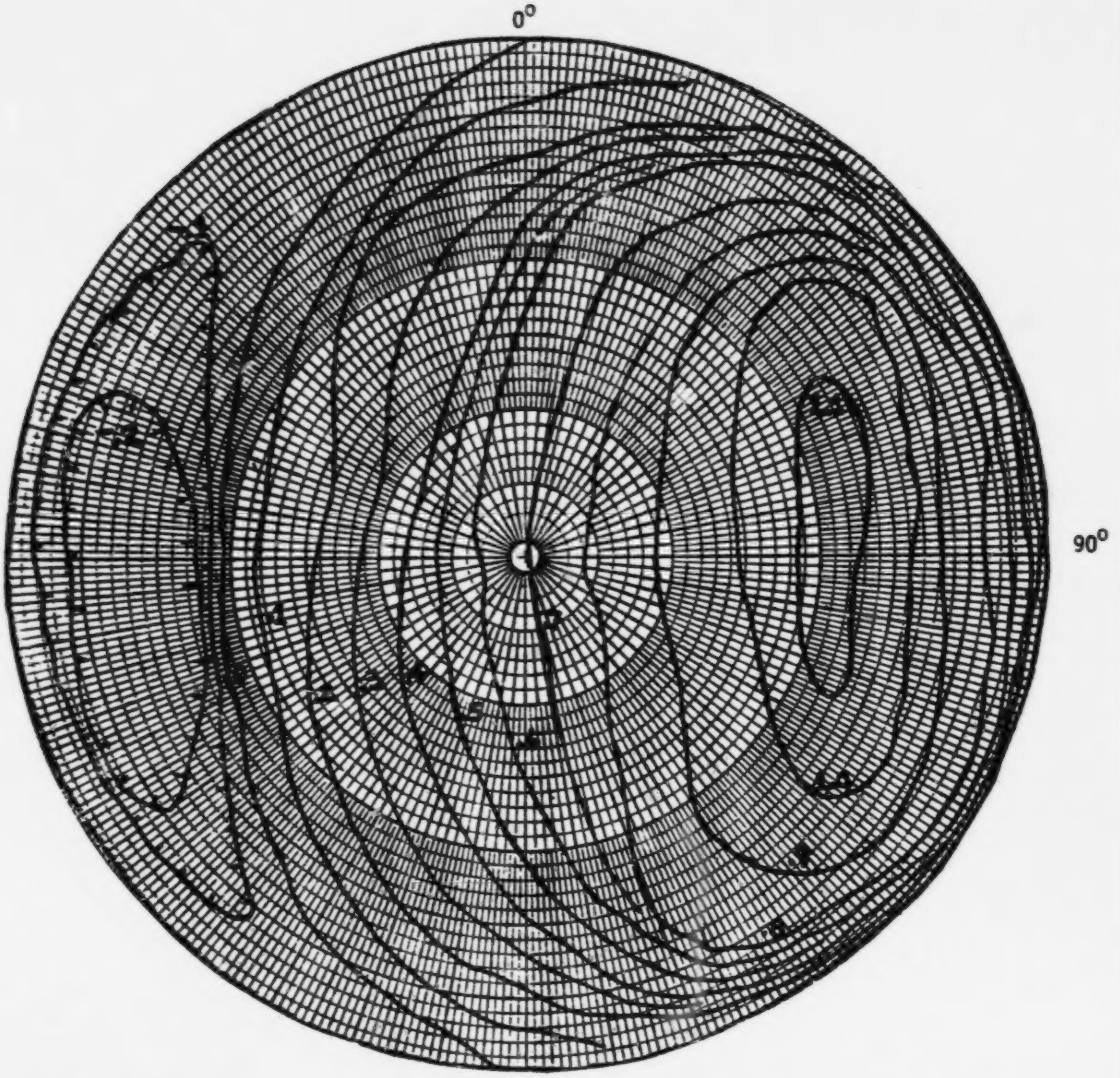


Figure 14a. Case VIII Hypothetical Velocity Profile
20' Diameter Duct

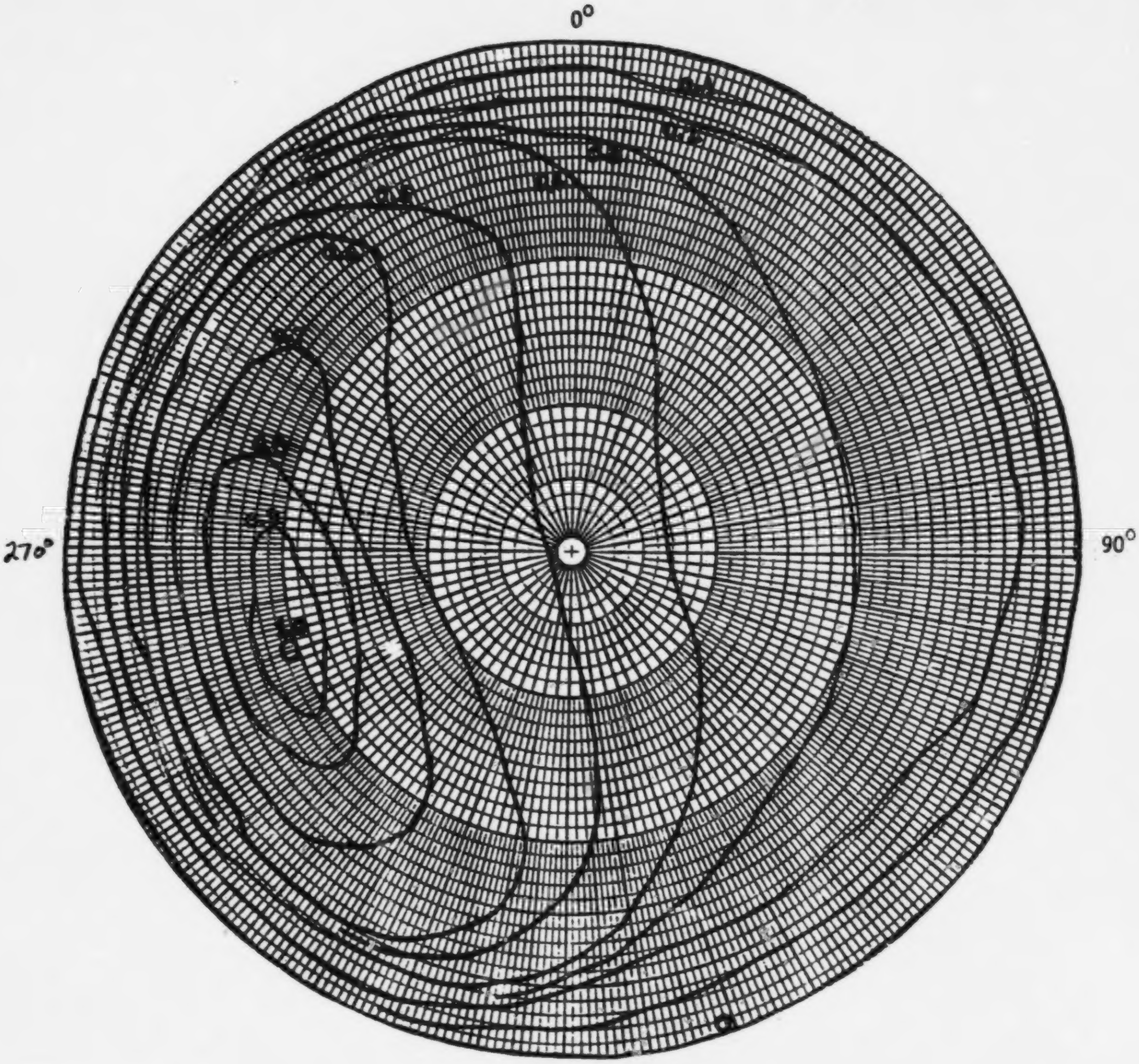


Figure 14b. Case VIII Hypothetical Concentration Profile
20' Diameter Duct

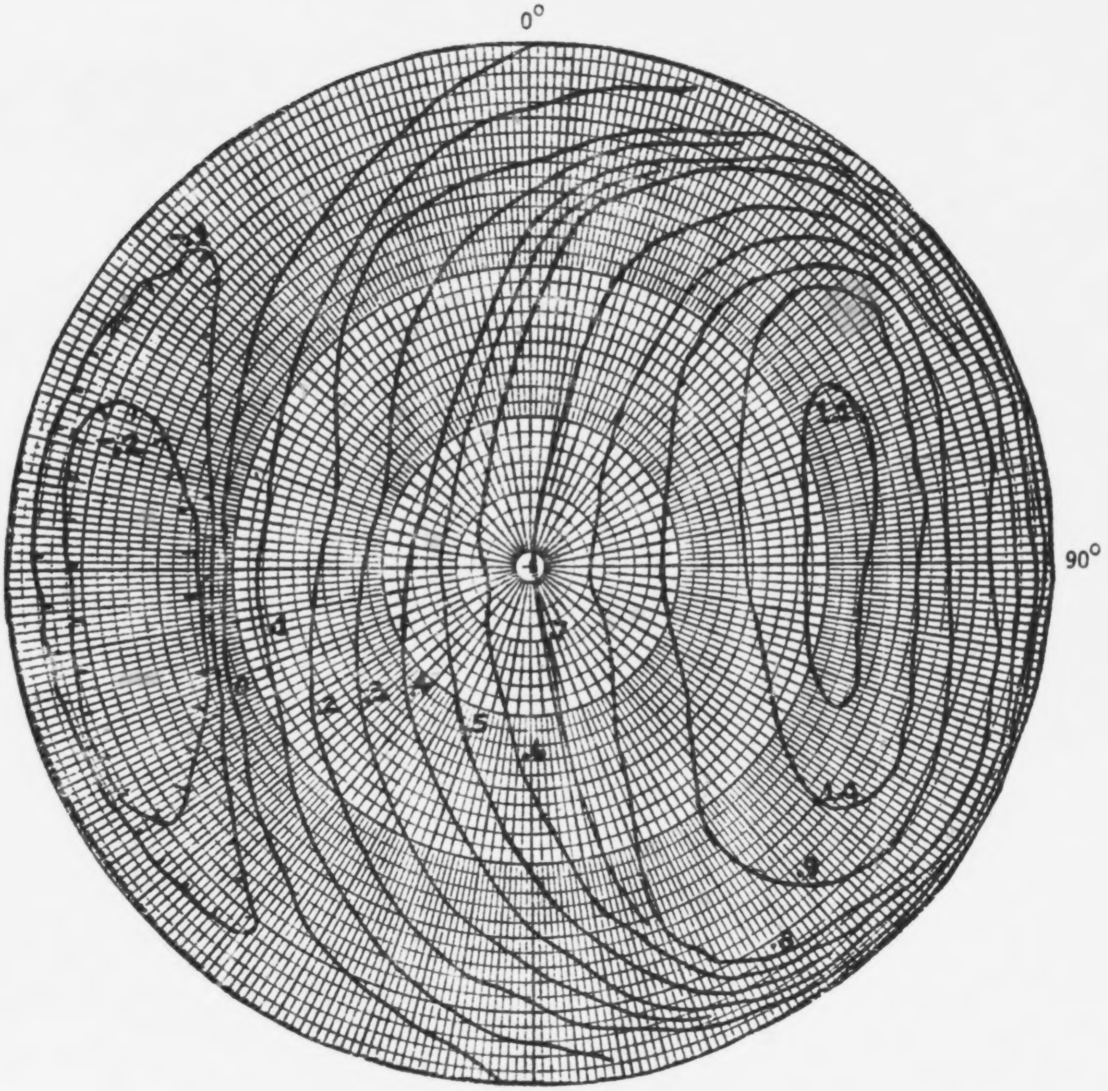


Figure 15a. Case IX Hypothetical Velocity Profile
20' Diameter Duct

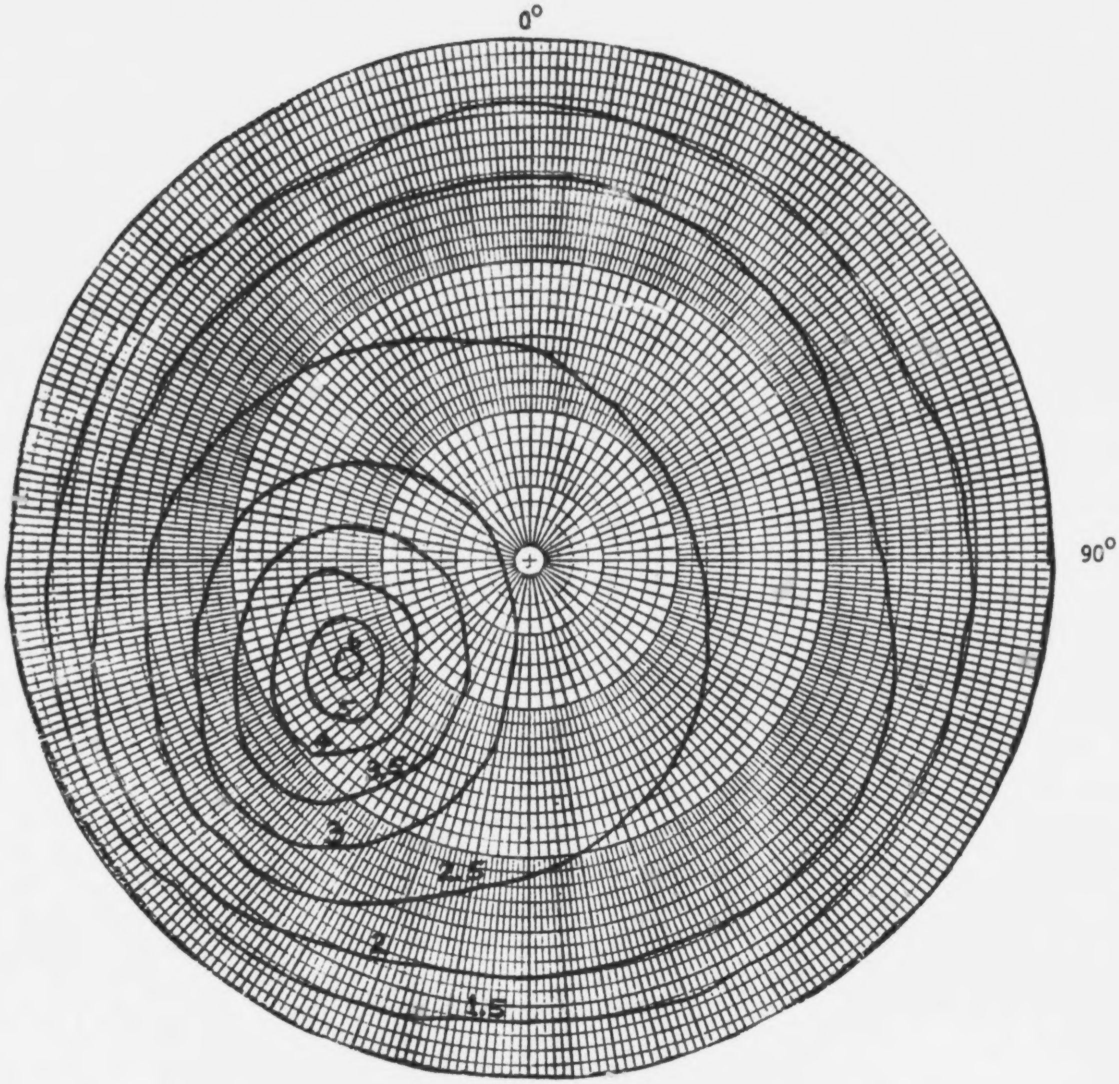


Figure 15b. Case IX Hypothetical Concentration Profile
20' Diameter Duct

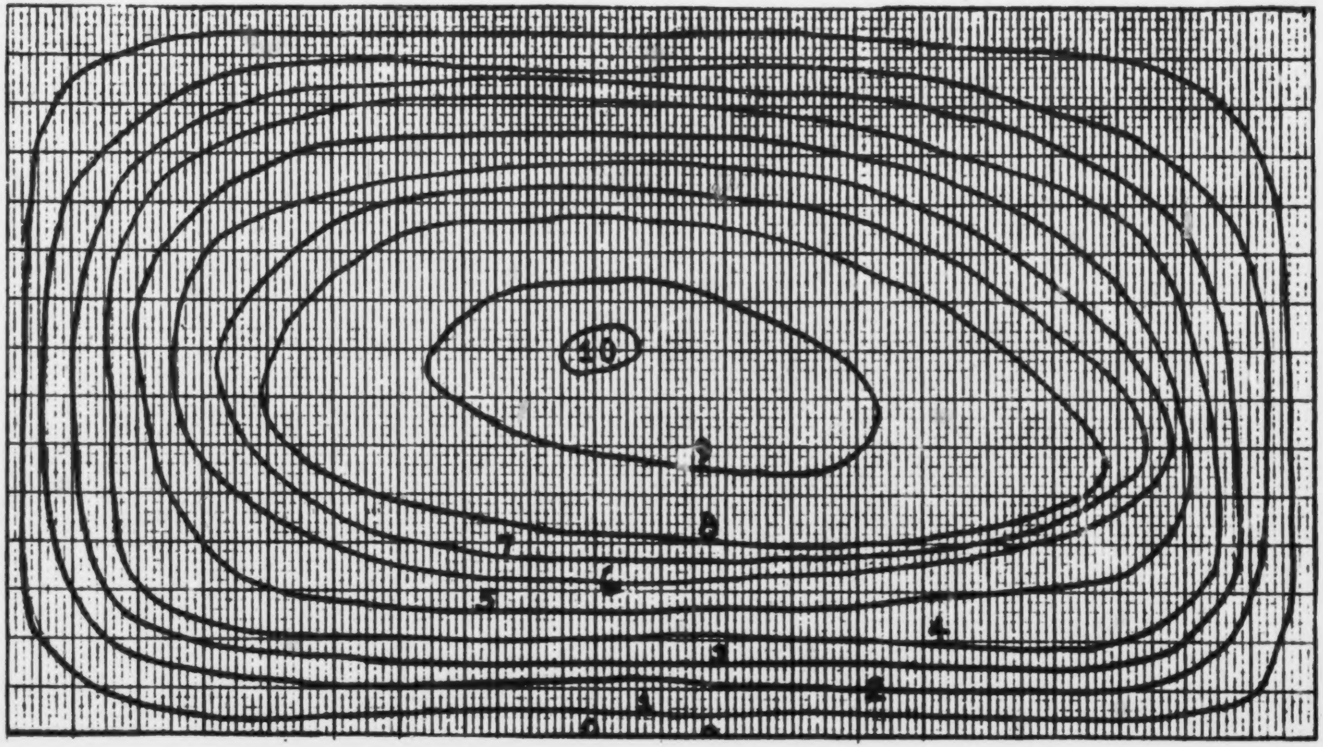


Figure 16a. Case X Hypothetical Velocity Profile
15' x 20' Duct

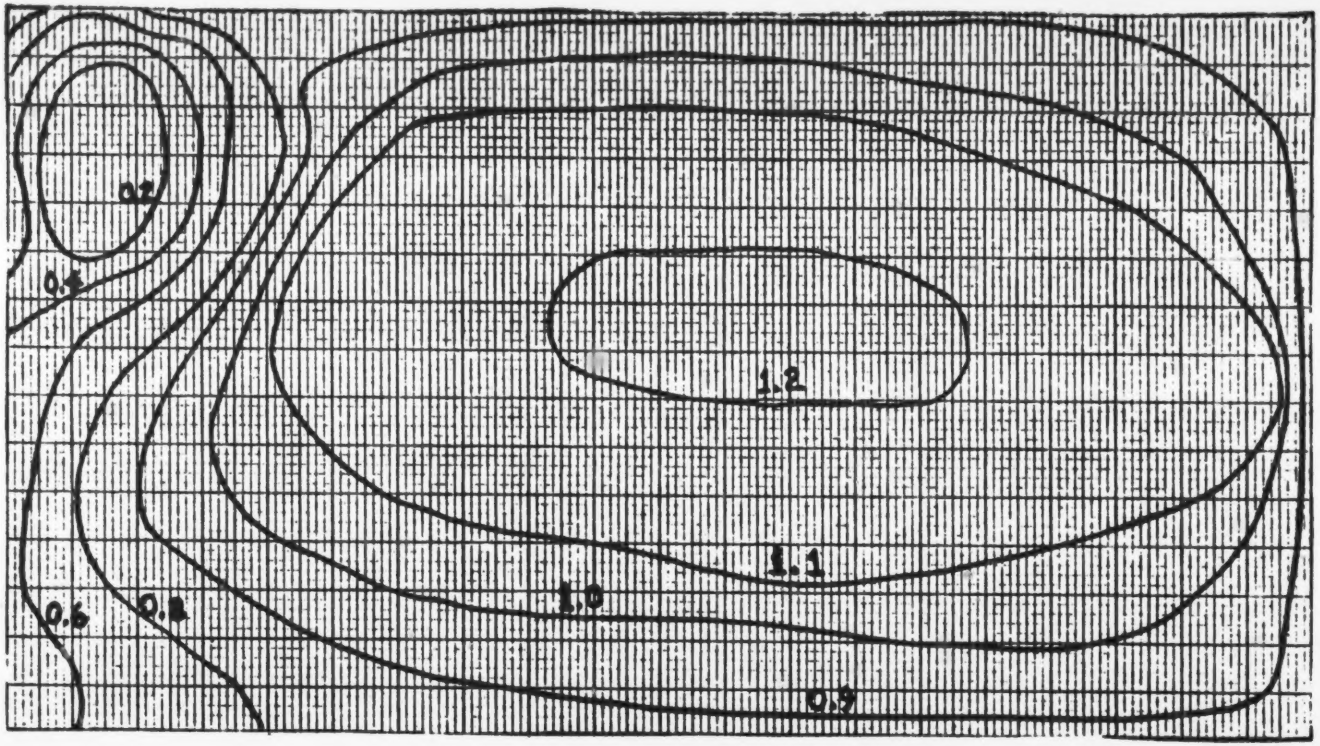


Figure 16b. Case X Hypothetical Concentration Profile
15' x 20' Duct

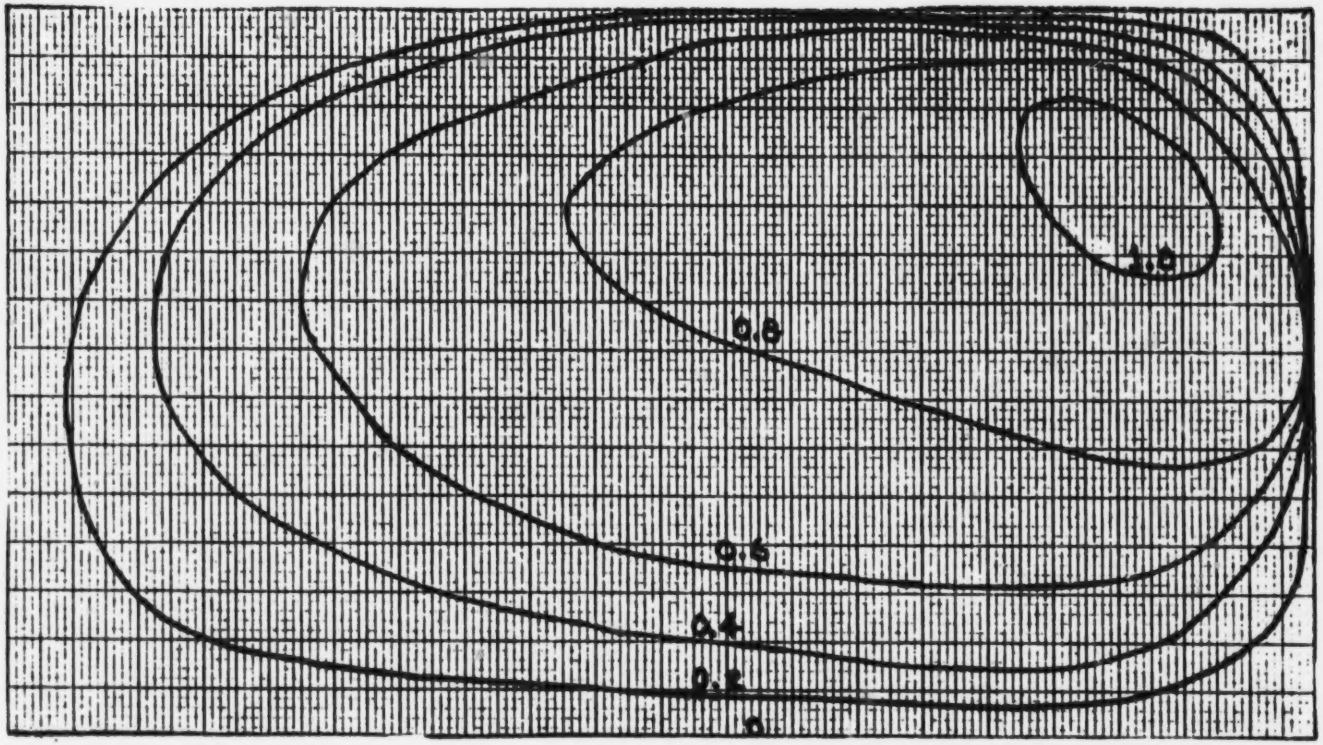


Figure 17a. Case XI Hypothetical Velocity Profile
15' x 20' Duct

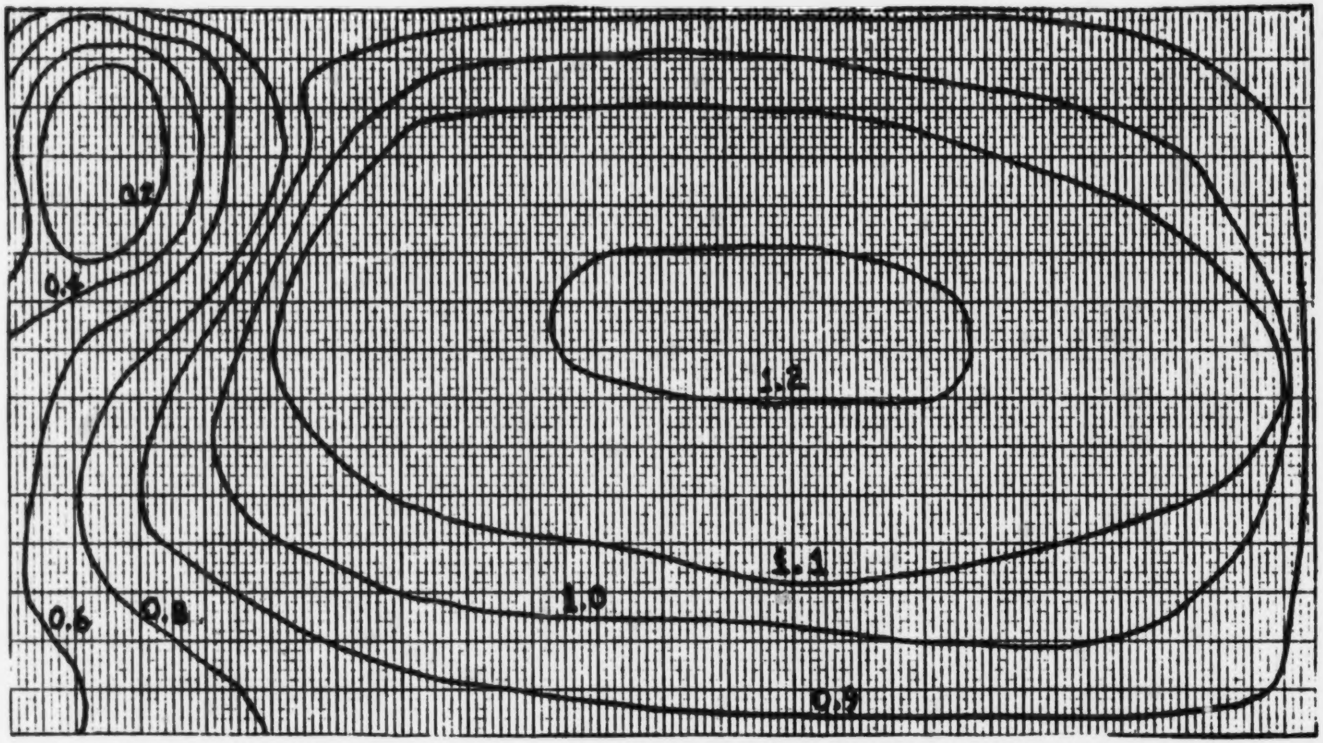


Figure 17b. Case XI Hypothetical Concentration Profile
15' x 20' Duct

TABLE 6

CASE XII TEST NO. 9-1

VELOCITY AND CONCENTRATION DISTRIBUTION DATA IN A SQUARE SECTION

		1	1-2	2	2-3	3	3-4	4	4-5	5	5-6	6	6-7	7
A	V[1]	2.1	2.1	2.1	3.0	4.1	3.1	3.5	3.5	3.5	3.0	2.8	2.3	2.3
	C[2]	85	90	85	95	105	110	110	115	120	95	100	85	95
B	V	2.6	2.1	2.8	3.0	3.1	3.6	3.7	3.8	3.6	3.5	3.2	3.0	3.0
	C	95	95	106	115	120	120	120	115	110	90	85	65	75
C	V	2.8	2.8	2.6	3.1	3.1	3.8	4.0	3.9	4.1	4.2	4.1	4.3	4.0
	C	105	115	125	125	130	130	130	120	116	90	90	65	65
D	V	2.3	2.6	2.6	3.0	3.3	3.9	4.2	4.4	4.5	4.9	4.9	4.7	4.7
	C	120	115	125	125	130	125	120	105	105	80	70	60	55
E	V	2.3	2.3	2.8	3.0	3.3	3.4	3.8	3.9	4.4	4.6	5.1	5.0	4.7
	C	100	105	110	120	115	115	100	95	85	65	50	35	35
F	V	2.6	3.0	3.2	3.3	3.6	3.4	3.8	3.9	4.1	3.9	4.6	4.9	4.6
	C	80	80	85	90	90	100	105	90	70	60	45	30	25
G	V	2.9	3.5	4.0	4.0	4.0	4.0	4.0	3.9	3.6	3.6	3.8	3.8	3.8
	C	60	55	55	60	60	65	70	70	60	50	45	30	25

CONDITIONS: Baffle: Inclined 22.5° to horizontal
Ethane Flow Rate: 20.23×10^{-6} kg/s
Temperature: 20°C

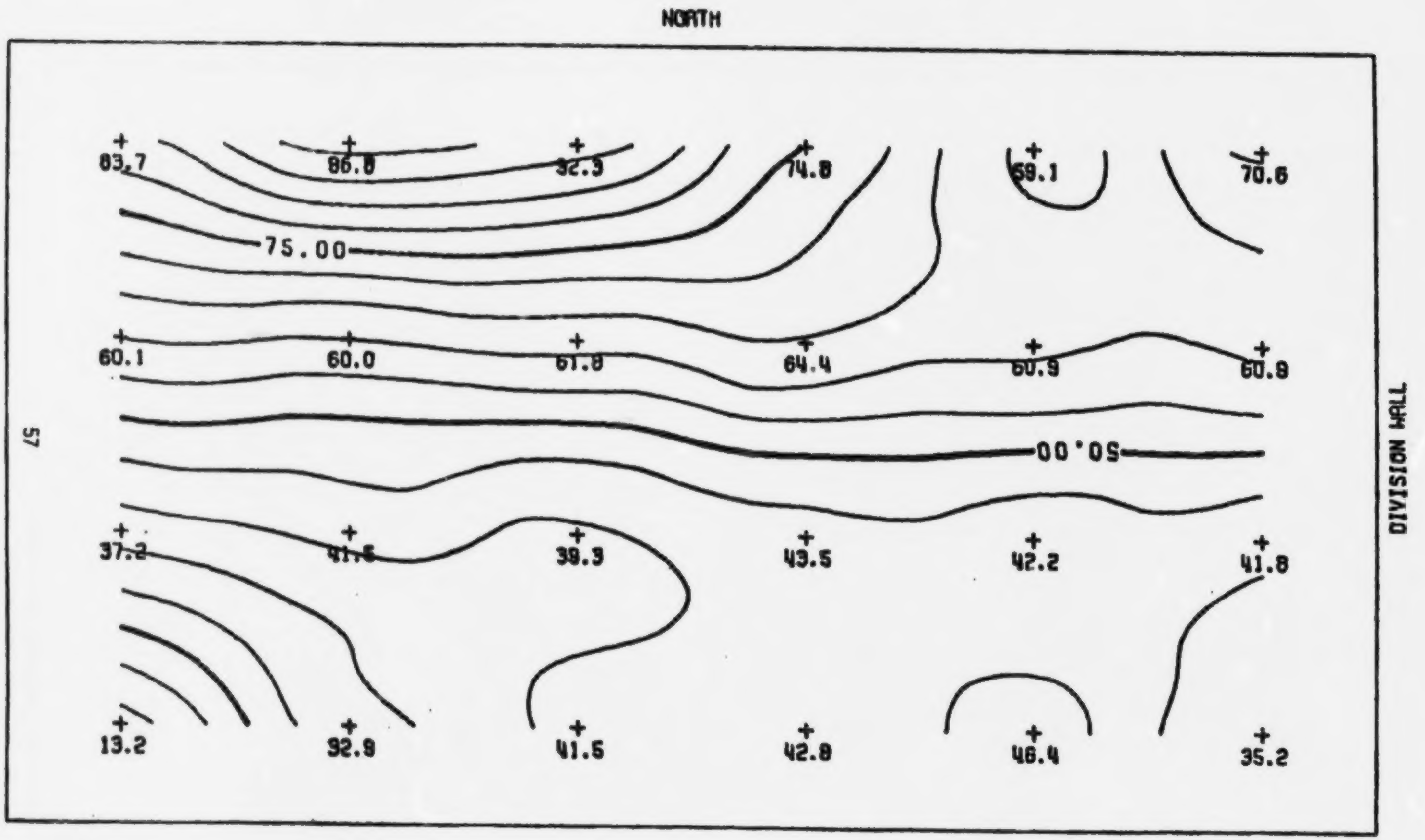
Flow Area: $0.18m^2$ ($280m^2$)
Number of Fans: 2 fans in series

---

[1] velocity m m/s

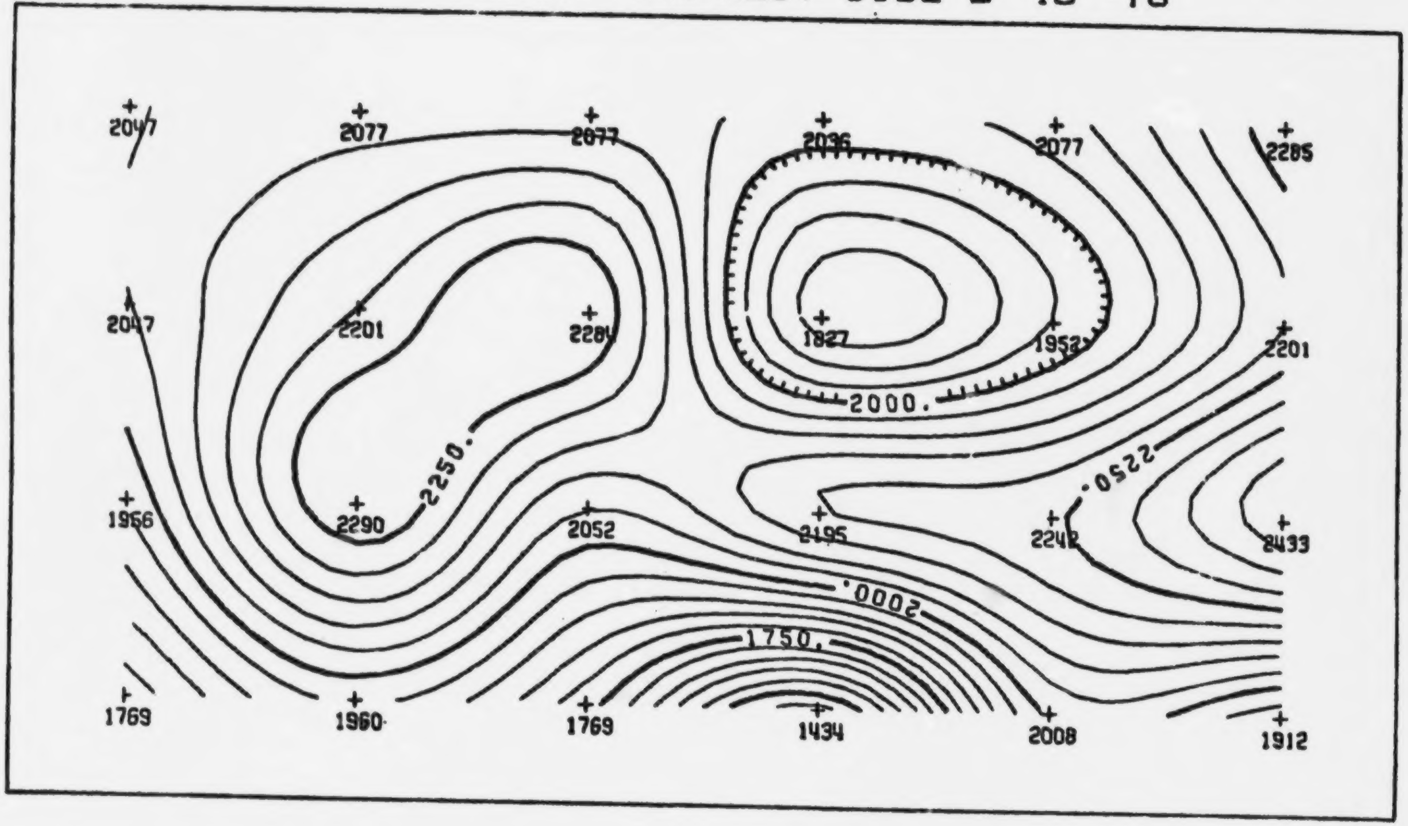
[2] concentration must be multiplied by 0.298 to convert to ppm ethane: Average Velocity = 3.5 m/s (11.6 ft/sec)
Total Flow = $0.64 m^3/s$ ($1356 ft^3/min$) at 20°C Total Emission = 20.62×10^{-6} kg/s

PLANE A-A WEST 4-7-70 RUN 6



ISØVELS - INTERVAL = 5 FT/SEC

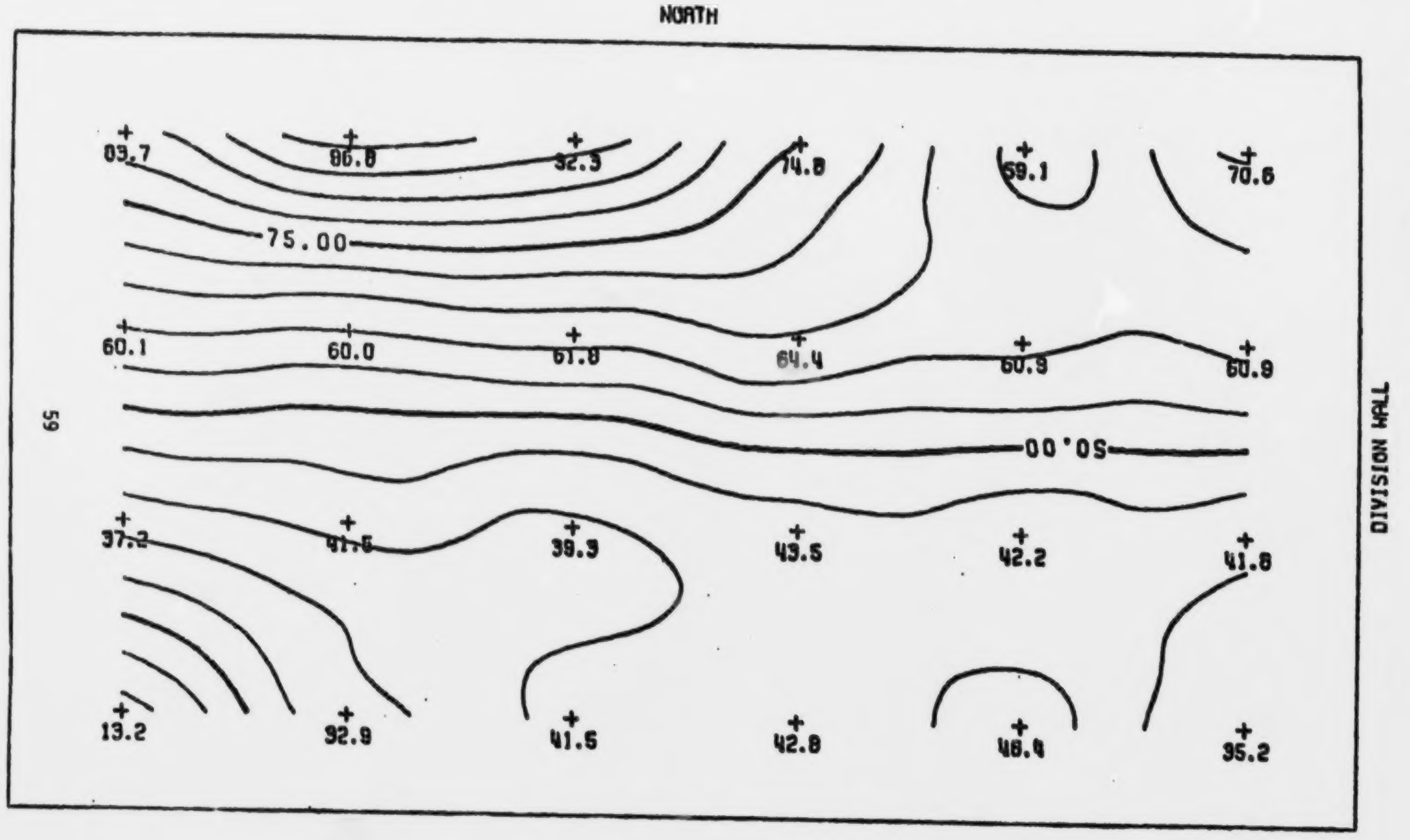
Figure 18a. CASE XIII-1


PLANE A-A WEST SIDE 3-16-70

S02 DISTRIBUTION - INT.=50 PPM

Figure 18b. CASE XIII-1

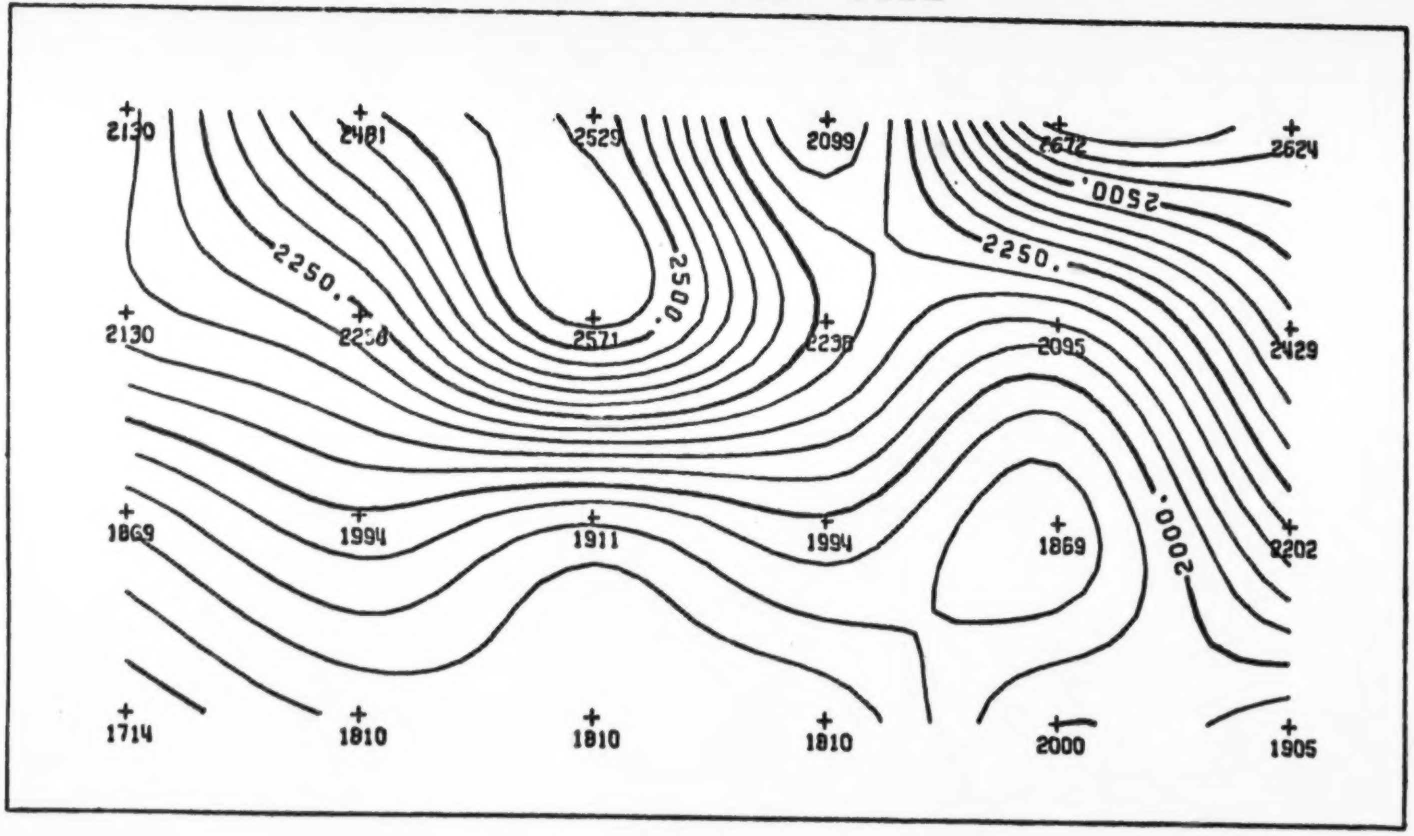
PLANE A-A WEST 4-7-70 RUN 6



ISOVELS - INTERVAL = 5 FT/SEC

Figure 19a. CASE XIII-2

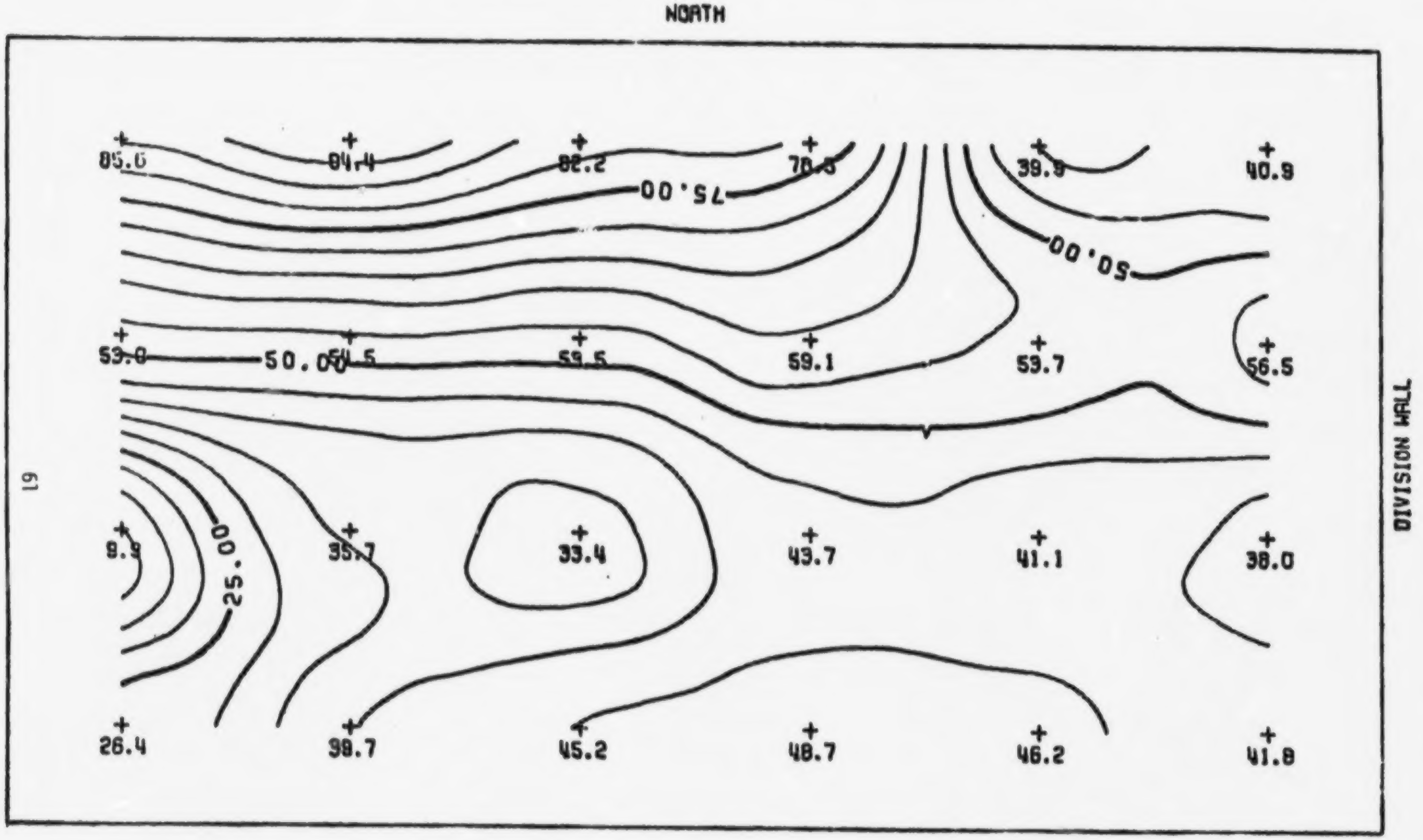
PLANE A-A WEST SIDE



SO2 DISTRIBUTION - INT.=50 PPM

Figure 19b. CASE XIII-2

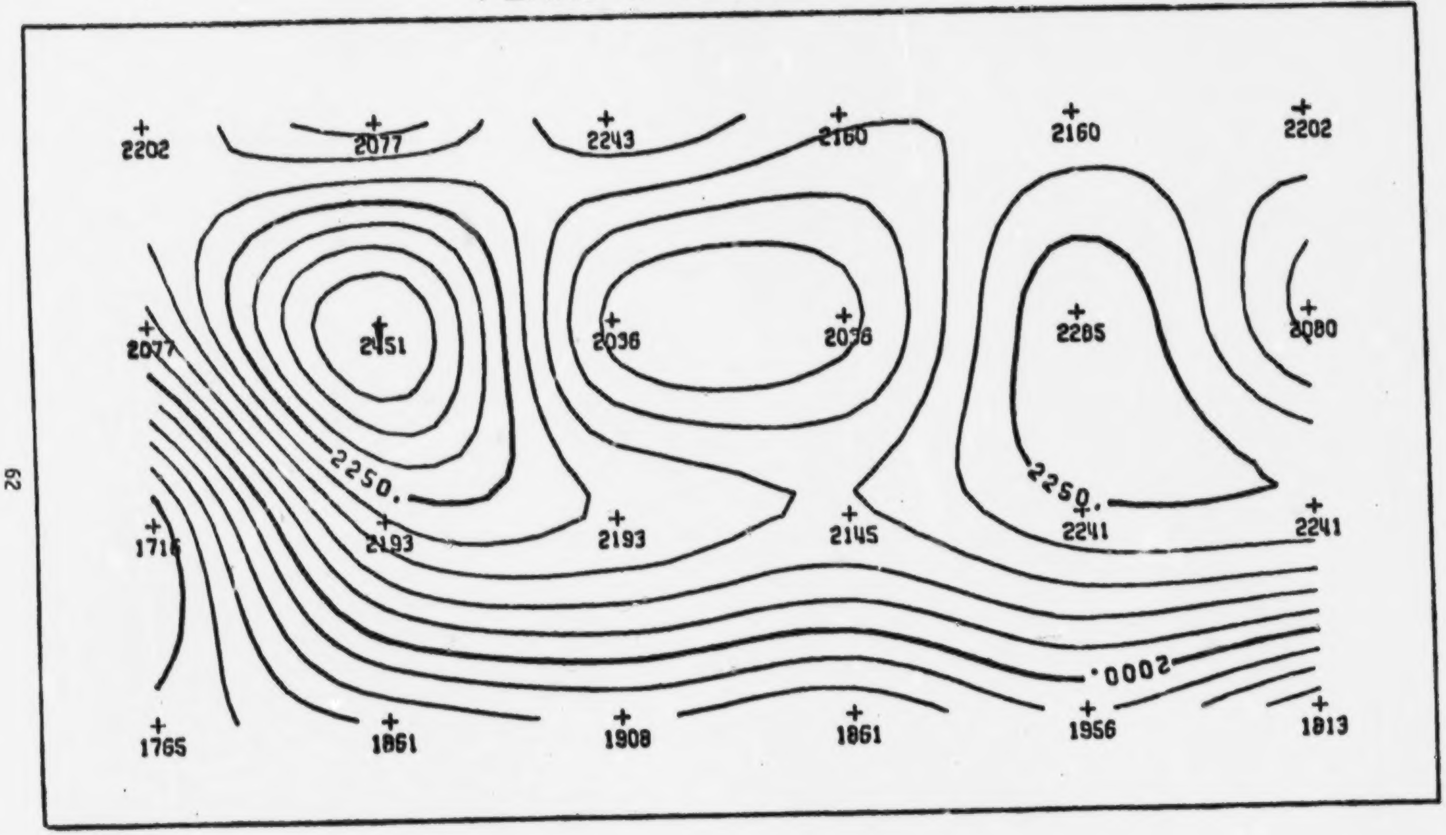
PLANE A-A WEST 4-7-70 RUN 5



ISØVELS - INTERVAL = 5 FT/SEC

Figure 20a. CASE XIII-3

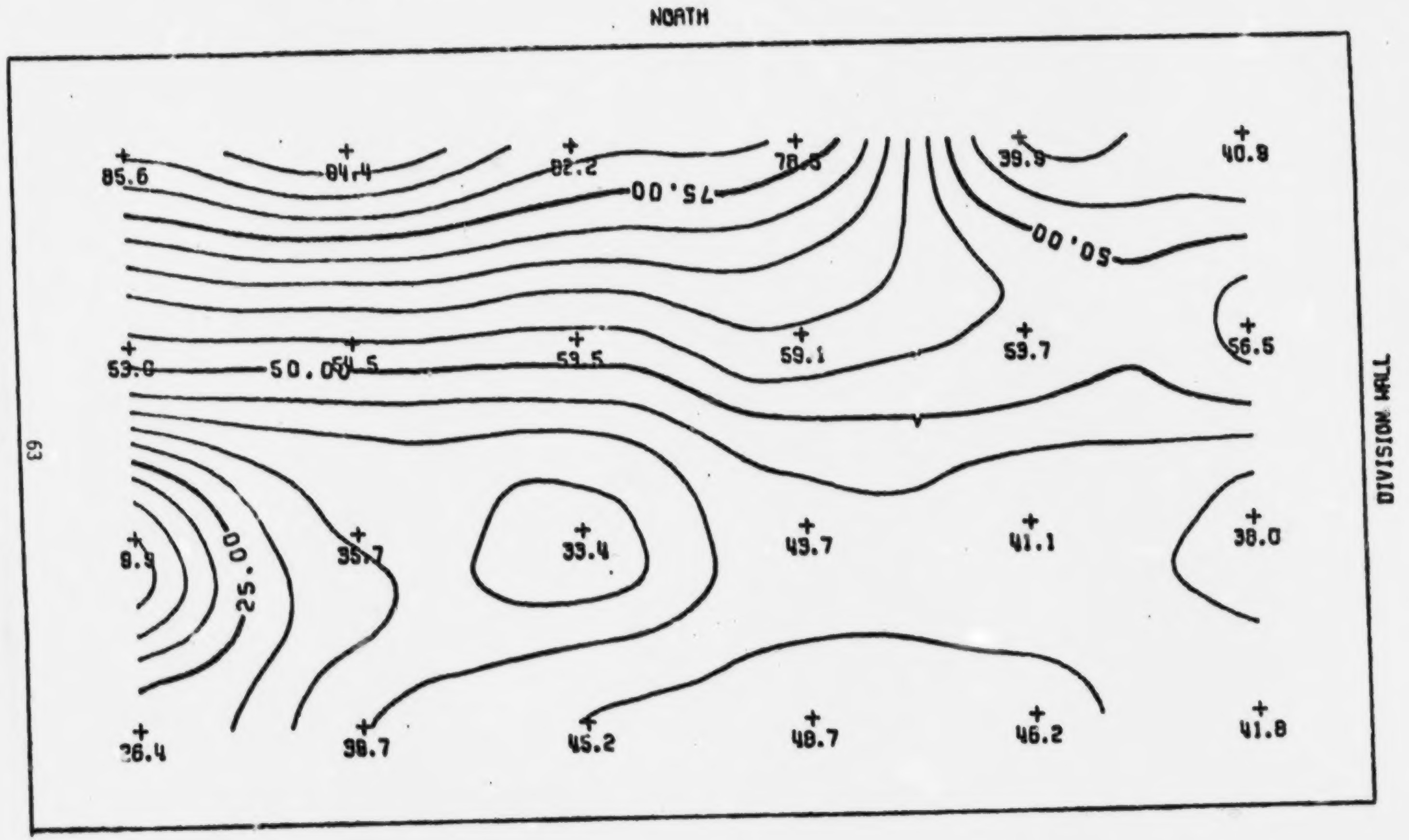
PLANE A-A WEST SIDE



SO2 DISTRIBUTION - INT.=50 PPM

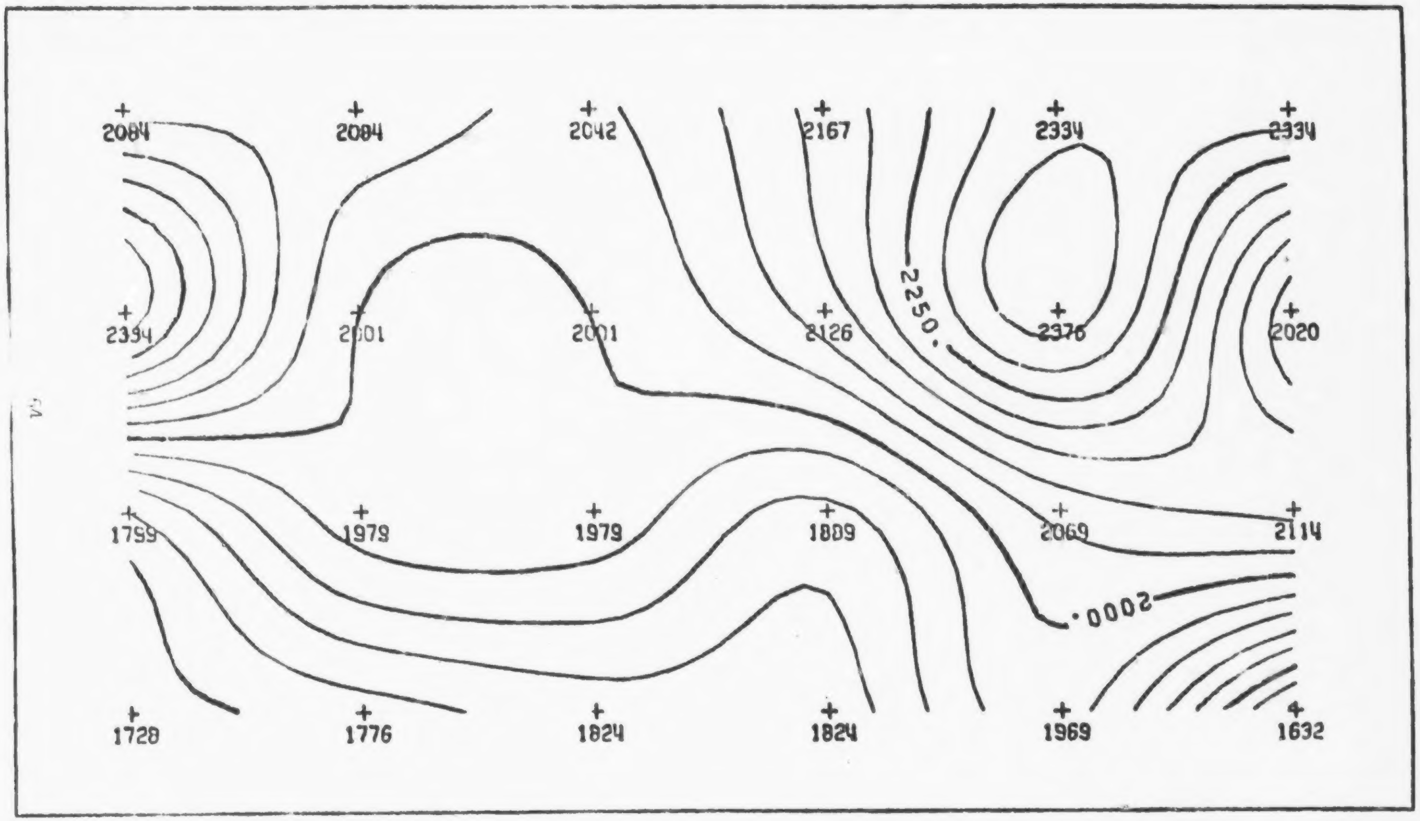
Figure 20b. CASE XIII-3

PLANE A-A WEST 4-7-70   RUN 5



ISOVELS - INTERVAL = 5 FT/SEC

PLANE A--A WEST SIDE



SO2 DISTRIBUTION - INT.=50 PPM

Figure 21b. CASE XIII-4

APPENDIX H. JET MIXING OF FLUE GAS STREAM

APPENDIX H-1

THE EFFECT OF JET IMPINGEMENT ON MIXING IN A GAS STREAM

A TWO DIMENSIONAL ANALYSIS OF THE RATE OF SPREADING OF A UNIT IMPULSE IN CONCENTRATION OF A GASEOUS SPECIES IN A FREE STREAM DUE TO JET IMPINGEMENT

By

Professor J. H. Porter, M.I.T., Consultant
Professor W. L. Harris, Sr., M.I.T., Consultant

I. INTRODUCTION

In the determination of the emissions rate of a contaminant species in a gas stream emitted from a stack, it is necessary to measure the velocity of the gas and concentration of the contaminant at several locations within a stack cross section with probes. The accuracy of the estimate of emissions rate will depend on the weighting strategy used to associate the individual probe readings with the appropriate fraction of stack cross sectional area. However, independent of probe strategy, with any fixed strategy the accuracy of the estimate will increase, as the number of sampling probes in the stack are increased. Unfortunately, the cost of the analysis increases with an increasing number of probes used in the determination. Thus, it is desirable to consider alternative means of improving the accuracy of emissions rate determination in stacks.

If a uniform concentration and velocity profile existed in a stack, one need only take a single measurement of concentration and velocity to determine the exact emissions rate. However, for a large number of reasons (e.g., leaks in stack brick work, uneven combustion rates of fuel in combustors, flow blockage and stagnation in ducts, etc.) rarely, if ever, will one find uniform velocity and concentration profiles in stacks. In addition, since the height to diameter ratios in stacks are relatively small (H/D <50) equilibrium profiles are established before the stack effluent is admitted to the atmosphere. Thus, if the limiting condition of using a single velocity and concentration probe to make an accurate determination of emissions rate is to be approached, external or internal devices must be placed in the stack to promote mixing normal to the direction of flow of inequalities in local velocities and concentration.

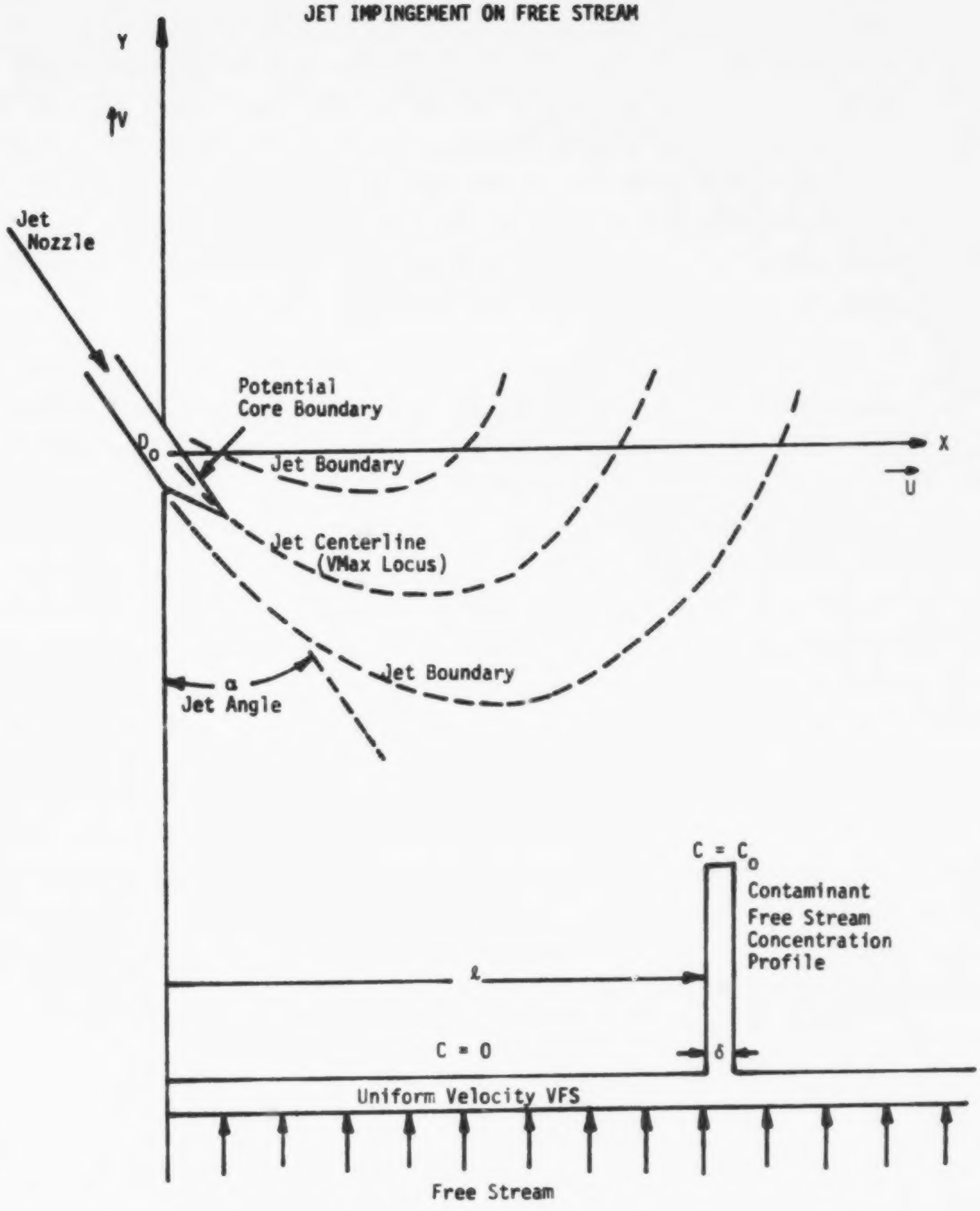
Several devices may be conceived to accomplish this task. The effectiveness of each device should be measured in terms of the rate of spreading induced by the device divided by the rate power is consumed by the device either from an external power supply or internally from the gas stream. This ratio (radial mass transfer rate/energy input rate) is at its maximum value for the most effective mixing device and diminishes with less effective devices. Thus for each device conceived this ratio should be determinable and compared to the value of this ratio for all other devices. In addition, the capital and annual operating cost of each device should be determinable and compared to the cost of other devices as well as probe capital and installation costs. Selection of devices should then be based on the cost of the total system, a device plus the number of probes required with each device in order to produce the same accuracy in emissions determination.

This report is concerned with the determination of the spreading ratio for a particular device, an induced jet impinging on the stack gas stream. The analysis because of complexity considers a two dimensional analysis of a jet stream which is compressed and directed at the stack gas. The stack gas is assumed to flow with uniform velocity but to contain a unit impulse in the concentration of a contaminant species at some distance ℓ from the entering jet. Radial diffusion in the free stream is assumed neglible. Diffusion in the jet is characterized as isotropic as induced by jet turbulence. The combination of the jet and free stream induce mixing of the contaminant. Figure 1 is a schematic of the model.

While a two dimensional analysis of the problem is not entirely accurate, its ease of solution relative to the three dimensional problem, and the consideration that the appropriate factors which cause spreading are included in the two dimensional analysis, dictates its choice as a model.

FIGURE H-1

JET IMPINGEMENT ON FREE STREAM


Y
V
Jet
Nozzle
Potential
Core Boundary
D_o
Jet Boundary
X
U
Jet Centerline
(VMax Locus)
Jet Boundary
α
Jet Angle
C = C_o
Contaminant
Free Stream
Concentration
Profile
ℓ
C = 0
δ
Uniform Velocity VFS
Free Stream

II. BACKGROUND

An essential objective of the proposed model is to determine a species concentration distribution and the axial rate of change of this distribution corresponding to an imposed velocity field and specified boundary conditions. Hence, the reliability of the results obtained from the proposed model is greatly enhanced by an accurate and logically self-consistent description of the jet properties and the velocity distribution along the intersection of the jet and free stream flow. The particular jet properties of interest are:

(1) The position of the jet centerline (Locus of points of maximum axial jet velocity)
(2) The spread (cross section growth) of the jet
(3) The velocity distribution in the jet
(4) The turbulent (eddy) diffusion coefficient distribution in the jet.

Each of the properties of the jet which are identified above vary with location, as there is a sequential pattern of development of a jet in a cross flow. This sequential pattern of development has been discussed by several investigators [1,2,3,4] and is briefly discussed here.

The jet has three regions with no sharp boundaries between the regions. The first region is identified as the potential core which exists before the turbulent shear region, generated along the jet boundary, extends to the centerline of the jet. Adjacent to the potential core is the zone of maximum deflection. The high intensity of shear at the jet-free stream boundary entrains the free stream rapidly into the jet and causes the flow direction of the jet to bend most rapidly to approach the flow direction of the free stream. The zone of maximum deflection is followed by the vortex zone in the far field where the flow is governed mainly by the motion of a turbulent vortex pair formed downstream of the bending over region. The degree of entrainment (mixing of the free stream with the jet fluid) increases monotonically and non-linearly from the potential core, through the zone of maximum deflection, to the vortex zone.

The required distribution of the jet properties of interest will be obtained by least-square fitting these properties from experimental measurements [43, 44, 45, 46]. Contained in references 43, 44, 45 and 46 are the required distributions of the jet properties of interest for a range of values of the ratio entering jet velocity to free stream velocity and angle of jet inclination relative to the free stream. The jet Reynolds number dependence has been shown to be negligible [44].

Consistent with the proposed two-dimensional model, the velocity distribution along the intersection of the jet flow and free stream flow will be determined by vectorial addition of the planar velocities involved. It is noted, however, that since the actual jet contains a pair of vortices downstream of the potential core, the velocity field is locally three-dimensional along the intersection.

III. MODEL DEVELOPMENT

The model to be developed in this section considers the velocity and turbulent diffusion coefficient distribution profiles developed by the interaction between an impinging jet and a free stream, and the resultant mixing in the free stream created by this interaction.

It is assumed that the system is isothermal and isobaric at temperature T and pressure P. At a sufficiently far distance upstream of the jet entry, so as not to be disturbed by the jet, flows a free stream with a uniform velocity in the X-direction from the jet entry exists a concentration pulse of some contaminant species of height C° and width δ in the free stream. If the width of the free stream flow channel is L (L is sufficiently large as not to effect the path of the jet) the mean concentration of the contaminant species is given by,

$$C = C^\circ \delta/L \tag{1}$$

and the contaminant emission rate is,

$$Q_s = V_s C^\circ \delta \tag{2}$$

The entrance velocity of the jet is V_o emitted uniformly through a planar nozzle of width D_o at an angle of inclination α with respect to the negative y direction ($\alpha = 0$).

Thus assuming an ideal fluid, the mass flow rate of jet fluid entering the free stream is:

$$Q_o = \rho V_o D_o = \left(\frac{MP}{RT}\right) V_o D_o \tag{3}$$

A. POWER REQUIREMENTS OF JET

Focusing attention first on the jet, the power requirements are computed by assuming a stagnant fluid at pressure P and T is adiabatically compressed to a pressure P^o, cooled at T, and expanded in a frictionless channel to velocity V_o and pressure P. The energy input rate for compression is given by:

$$E = \frac{Q_o RT}{M \gamma n} \left\{ \left(\frac{P^o}{P} \right)^{\gamma} - 1 \right\} \qquad (4)$$

Free expansion (isothermal) in a frictionless duct is given by Bernoullis equation which when integrated produces:

$$\left(\frac{P^o}{P} \right)^{\gamma} = \exp \left\{ \frac{\gamma M V_o^2}{2RT} \right\} \qquad (5)$$

Thus combining (3), (4) and (5):

$$E = \frac{P V_o D_o}{\gamma\, n} \left\{ \exp \left\{ \frac{\gamma M V_o^2}{2RT} \right\} - 1 \right\} \qquad (6)$$

B. MASS TRANSPORT

Considering an area element dx by dy in the plane of the jet and the free stream the steady state mass conservation equation for any species is:

$$\frac{\partial}{\partial x}\left\{E(x,y)\frac{\partial c}{\partial x} - U(x,y)C\right\} + \frac{\partial}{\partial y}\left\{E(x,y)\frac{\partial c}{\partial y} - V(x,y)C\right\} = 0 \qquad (7)$$

where $E(x,y)$ is the diffusion coefficient and $E(x,y)$ outside the jet boundaries is zero

$U(x,y)$, $V(x,y)$ are the x and y components of the velocity vector

with the initial conditions:

$$U(x,-\infty) = 0 \qquad U(o,y) = 0$$

$$V(x,-\infty) = V_s \qquad \frac{\partial V}{\partial y}(x,-\infty) = 0$$

$$\left.\begin{array}{lll} C(x,-\infty) = 0 & x<\ell \\ C(x,-\infty) = 0 & x>\ell+\delta \\ C(x,-\infty) = C^o & \ell\leq x\leq\ell+\delta \end{array}\right\} \quad \frac{\partial c}{\partial y}(-x,-\infty) = 0 \qquad \frac{\partial c}{\partial x}(o,y) = 0$$

The equations describing the velocity and diffusion field $\left(U(x,y), V(x,y), E(x,y)\right)$ will be developed based upon experimental results on jets injected into cross flow streams. The solution to equation (7) (carried out numerically) will provide concentration profiles in x and y.

C. SPREADING COEFFICIENT

In a completely mixed stream the concentration is uniform and equal to the average concentration:

$$\bar{C} = C^{o}\delta/L$$

The deviation of the point concentration from the average, $\bar{V}^2(x,y)$ will be defined as,

$$\bar{V}^2(x,y) = \left(C(x,y) - \bar{C}\right)^2 \tag{8}$$

and the average value at any plane of constant y as

$$\bar{\bar{V}}^2(y) = \frac{1}{L}\int_0^L \left(C(x,y) - \bar{C}\right)^2 dx \tag{9}$$

Initially, $\bar{\bar{V}}^2$ has a value,

$$\bar{\bar{V}}^2(-\infty) = \frac{1}{L}\left\{\int_0^{\ell} (o-\bar{C})^2 dx + \int_{\ell}^{\ell+\delta} (C^{o} - \bar{C})^2 dx + \int_{\ell+\delta}^{L} (o - \bar{C})^3 dx = C^{o^2} \left(\frac{\delta}{L}\right) \left(1 - \frac{\delta}{L}\right)\right.$$

The rate of change of $\bar{\bar{V}}$ will be defined as the spreading coefficient:

$$\mathcal{S} = \frac{d\bar{\bar{V}}(y)}{dt} = V_s \frac{d\bar{\bar{V}}(y)}{dy} = V_s \frac{d}{dy} \left\{\sqrt{\frac{1}{L}\int_0^L \left(C(x,y) - \bar{C}\right)^2 dx}\right\} \tag{10}$$

and its average value over some axial channel length, H defined by:

$$\bar{\mathcal{S}} = \frac{1}{H}\int_{-\infty}^{H} \mathcal{S}(y)\,dy = \frac{V_s}{H}\int_{-\infty}^{H} \frac{d}{dy} \left\{ \sqrt{\frac{1}{L}\int_0^L \left(C(x,y) - \bar{C}\right)^2 dx} \right\} dy \tag{11}$$

The average speading coefficient divided by the jet energy input will be the rating factor to measure the effectiveness of jet mixing

$$\text{r.f.} = \bar{\Delta}/E \qquad (12)$$

It is noted that

$$\text{r.f.} = \frac{\bar{\Delta}}{E} = \frac{\frac{V_s}{H}\left\{\sqrt{\frac{1}{L}\int_0^L \left(C(x,H) - \bar{C}\right)^2 dx} - C^0\sqrt{\frac{\delta}{L}\left(1 - \frac{\delta}{L}\right)}\right\}}{\left\{\frac{PV_oD_o}{\gamma\,\eta}\left[\exp\left(\frac{\delta MV_o^{\,2}}{2RT}\right)-1\right]\right\}} \qquad (13)$$

D. VELOCITY AND TURBULENT DIFFUSION PROFILES WITHIN A JET

The development of correlations to predict jet properties is based primarily upon experimental evidence [43, 44, 45, 46]. These results are presented herein.

The ratio of the entrance jet velocity to the free stream velocity is defined as λ ,

$$\lambda = \frac{V_o}{V_s}$$

Referring to Figure 2, if Z is the y location of the jet centerline, then Z's dependence on x, λ and a is given by:

$$\frac{Z}{D_o\lambda} = .2293 \left(\frac{X}{D_o \lambda\sin\alpha}\right)^{2.6316} - \left(\frac{X \text{ cat } \alpha}{D_o\lambda}\right) \qquad (14)$$

If R is the radius of the jet, R is given by:

$$\frac{R}{\lambda D_o} = .46 \left(\frac{Z}{\lambda D_o}\right)^{.4} \qquad (15)$$

If ζ and η are the coordinate axes in the Z and R directions respectively, the velocities in these directions are given by $V(\eta,\zeta)$ and $U(\eta,\zeta)$ respectively. Thus, the velocities within the jet in the x and y directions are given by the relationships,

$$U(x,y) = V(\eta,\zeta) \cos \beta - U(\eta,\zeta) \sin \beta \qquad (16)$$

$$V(x,y) = V(\eta,\zeta) \sin \beta - U(\eta,\zeta) \cos \beta \qquad (17)$$

where β is defined by:

$\text{Tan } \beta = \frac{dz}{dx}$ (18) and $Z = f(x)$ is given by equation (14)

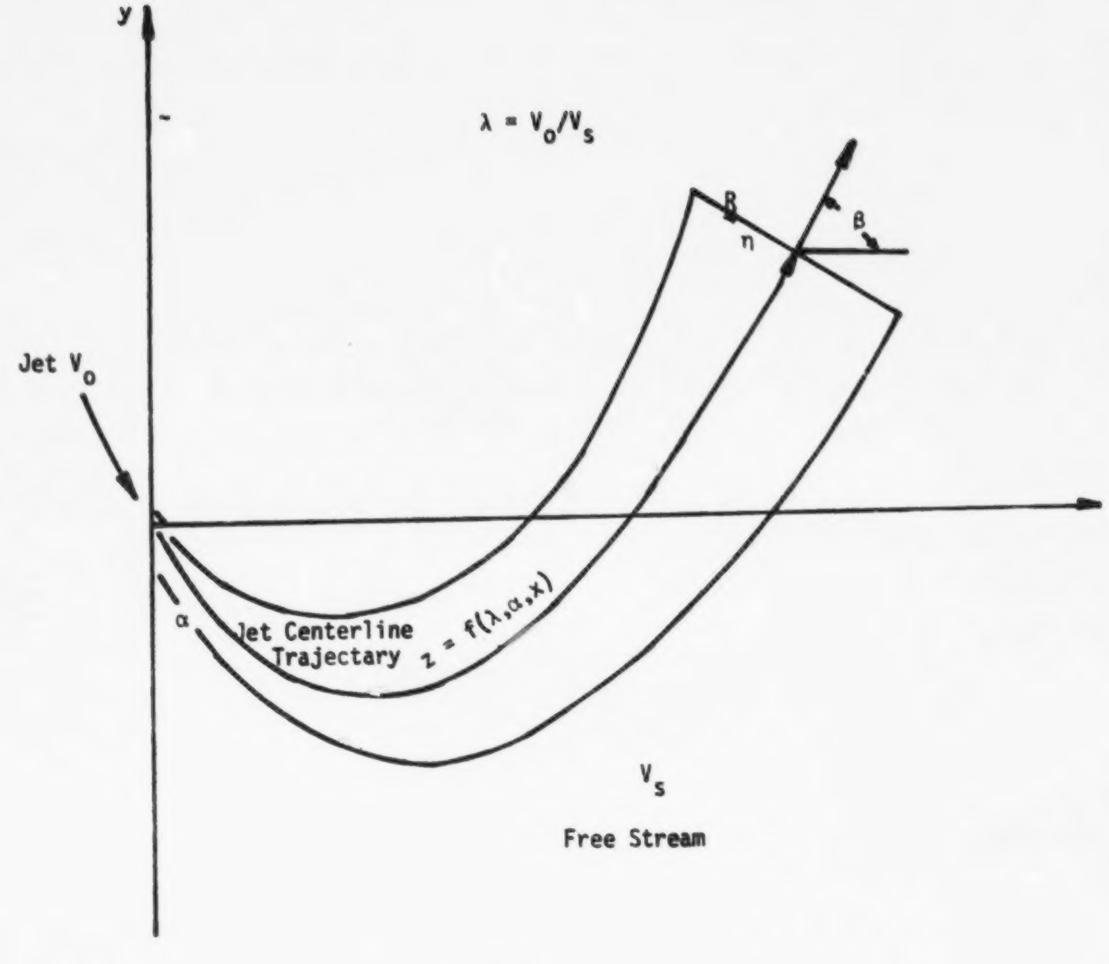

y
λ = V_o/V_s
β
η
β
Jet V_o
α
Jet Centerline
Trajectary
z = f(λ,α,x)
V_s
Free Stream

FIGURE 2

POSITION OF JET CENTERLINE

Since velocity profiles within the jet are given with respect to the η, ζ axes, these can be coonverted to velocities parallel to the x and y axes by the above expressions.

The η, ζ components of the velocity are given by,

$$U(\eta, \zeta) = 0 \tag{19}$$

and $V(\eta, \zeta)$ is obtained by correlating the data of Keffer and Baines [1] shown in Figure 3, and plotted as:

$$\frac{V(\eta, \zeta) - V_s}{V_{(o, \zeta)} - V_s} = f\left(\frac{\eta}{\eta^*}\right) \tag{20}$$

where η^* is defined as the location in the jet where the velocity falls to one half the center line value. Correlations of η^* and $V(o, \ )$ are shown in the same figure.

The eddy diffusion coefficient ε is given in terms of an entrainment coefficient E as

$$\varepsilon_{(\zeta)} = \frac{R}{2\sqrt{\pi}} \left(\overline{V}_{(\zeta)} - V_s \right) E \tag{21}$$

where $\overline{V}_{(\zeta)} = \int_0^R 2\pi\eta V(\eta, \zeta) d\eta \Big/ \pi R^2$

Since $V(\eta, \zeta)$ is given by (20).

The diffusion coefficient, only dependent upon z and uniform in the η direction is determinable if E is known. Figure 4 is a correlation for E.

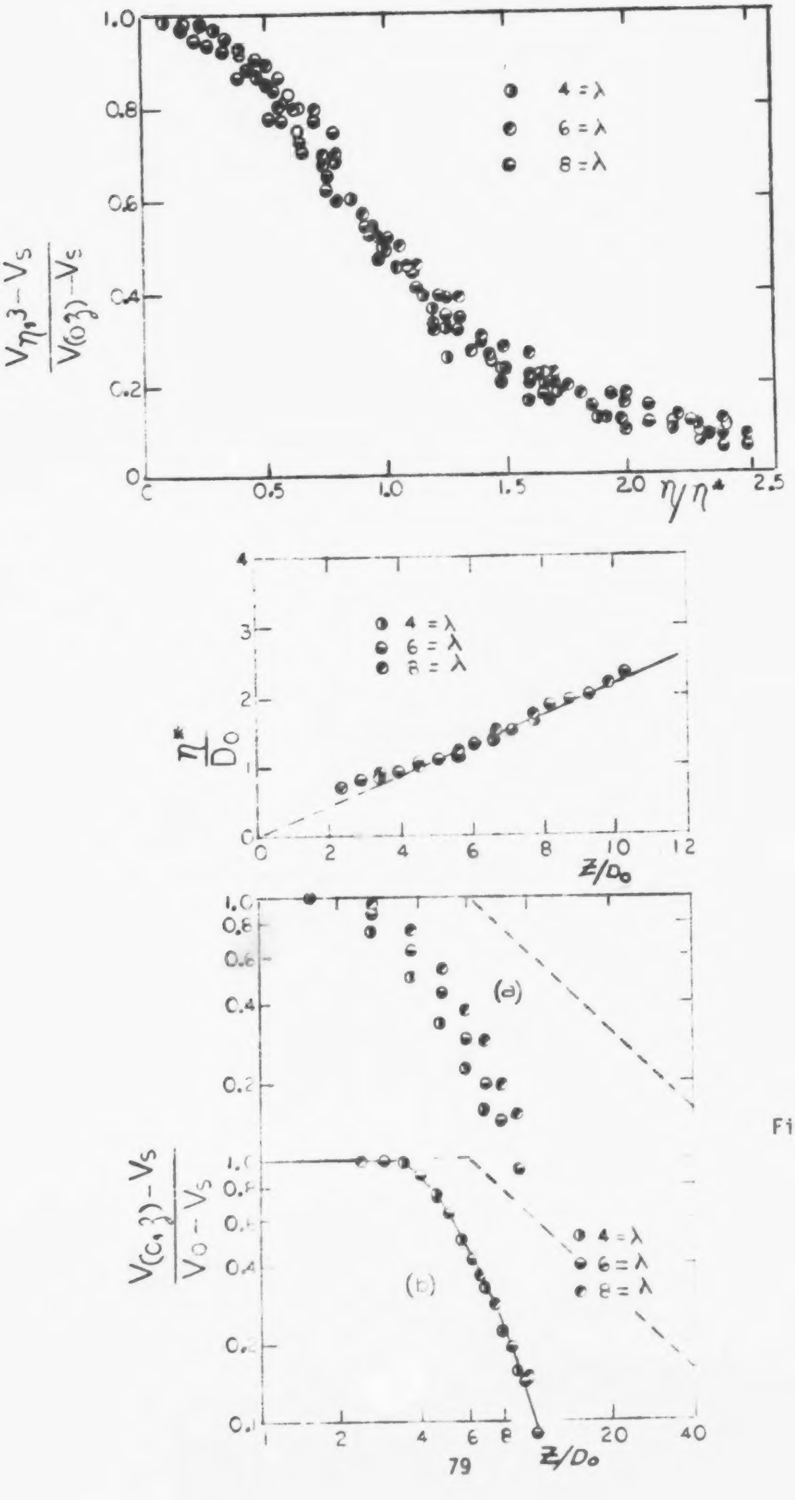
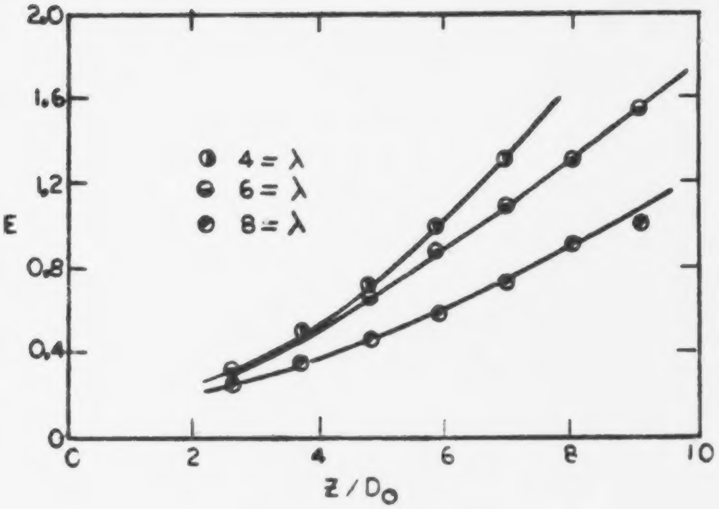

$\frac{V_{\eta,z} - V_s}{V(0,z) - V_s}$
1.0
0.8
0.6
0.4
0.2
0
0.5
1.0
1.5
2.0
2.5
η/η^*
4 = λ
6 = λ
8 = λ
$\frac{\eta^*}{D_0}$
4
3
2
1
0
2
4
6
8
10
12
Z/D_0
$\frac{V(0,z) - V_s}{V_0 - V_s}$
(a)
(b)
0.8
0.6
0.4
0.2
0.1
1
2
4
6
8
20
40
Z/D_0

Figure 3.

Figure 4.



The preceding description thus provides a mechanism to determine the velocity and diffusion in a jet.

IV PROPOSED SIMULATION

A solution of the partial differential equation governing the concentration field of the proposed model corresponds to solving the diffusion equation with variable coefficients, the description of the velocity and eddy diffusion coefficient fields. The numerical simulation of the partial differential equation governing the concentration will consist of an implicit finite difference model of the governing equations and boundary conditions. Suitable explicit finite difference models may be developed; however, due to the error propagation characteristics inherent to explicity finite difference approximations of parabolic equations, it is recommended that implicit schemes be exploited initially.

V. EXPECTED RESULTS

The numerical simulation of the equation describing mass transport will provide concentration profiles (c = f(x)) for all axial positions y along the duct. This information is then used to compute the local spreading coefficient,

$$\frac{d\bar{v}(y)}{dt} \equiv \frac{d}{dt}\left\{ \frac{1}{L}\int_0^L \left(C(x,y) - \bar{C}(y)\right)^2 dx \right\}^{\frac{1}{2}} = \frac{Vd}{dy}\left\{ \frac{1}{L}\int_0^L \left(C(x,y) - \bar{C}(y)\right)^2 \right\}^{\frac{1}{2}}$$

where $\bar{C}(y) = \frac{1}{L}\int_0^L C(x,y)dx$ and the axial average value $\int_0^H \frac{d}{dt}\bar{V}\,dy = \int_0^H V(y)\,\frac{dJ}{dy}\,dy$

The power required to produce the jet may be computed thus determining the effectiveness of jet mixing (the rate of mass transport in the X-direction divided by the rate of energy input. The functional dependence of this quantity on jet angle and distance of the initial concentration pulse from the jet wall may also be determined so as to estimate the best location of jet mixers relativ to the free stream.

VI. NOMENCLATURE

R.f.	Jet mixing rating factor - gr-cmv/ERG
$\bar{\sigma}$	spreading coefficient - gr/cc-sec
$U(x,y)$	x velocity component to fluid - cm/sec
$V(x,y)$	y velocity component to fluid - cm/sec
$E(x,y)$	Turbulent diffusion coefficient - cm^2/sec
α	Angle of jet inclination - radiants
D_o	Diameter of jet nozzle - cm.
ℓ	cm distance in the x-direction from the jet entry to the concentration pulse
L	Width of the free stream flow channel, L cm
δ	Width of concentration pulse - cm
C	Concentration - gm/cm^3
C°	Concentration pulse height - gr/cm^3
T	System temperature - °K
P	System pressure - dryness/cm^2
V_o	Jet entrance velocity - cm/sec
V_s	Uniform free stream velocity - cm/sec
$\overline{C}$	Average normal concentration - gm/cm
Q_s	Contaminant emissions rate - gr/xec - cm
M	Molecular weight of fluid - gr/gr-mole
R	Gas constant - erg/gr-mole °K
Q_o	Jet mass flow rate - gr/cm-sec
C_p	Fluid heat capacity at constant pressure - erg/gr °K
C_v	Fluid heat capacity at constant volume - erg/gr °K
X	C_p/C_v $(x-1)/K$
η	Compression efficiency
E	Rate of energy input to produce jet ergs/cm, EC
H	Axial height - cm
λ	V_o/V_s
z	Vertical distance of jet center line above jet entry - cm
$\dot{R}$	Jet Radius - cm
β	Angle between the locus of maximum jet velocity and the X axis - cm

APPENDIX H-2

JET MIXING IN A DUCT

Assumptions: [47, 48]

1. Jet is directed at right angles to main stream
2. Can neglect the effect of non-uniform velocity in main stream (assume slug flow)
3. Can neglect the effect of ambient turbulence on mixing near jet orifice
4. Jet is turbulent - $Re = \frac{du}{\nu} \gtrsim 300$.

nomenclature

d	orifice diameter (b = radius)
D	duct diameter
u	mean jet velocity at orifice
V	mean gas velocity in duct

For a turbulent jet ejected normal to a uniform cross flow, it is possible to define a momentum length scale

$$lm = d \frac{u}{V}$$

This represents the scale over which the vertical momentum of the entrained fluid becomes comparable to the initial momentum flux at the orifice. (in a rough sense it represents how far the fluid is projected into the crossflow within a few orifice diameters downstream.)

Consider now a jet in a duct of diameter D. It is possible to assure ourselves of geometric similarity by making $lm/_D$ fixed. Thus, set

$$\frac{lm}{D} = \frac{du}{DV} = 1 \quad ; \frac{d}{D} < 1 \qquad (1)$$

which defines a well mixed jet. (It is possible to show that this relationship will guarantee a local jet width comparable to the duct diameter.) (It is also possible to show that equation 1 assures mixing within a few duct diameters from orifice.)

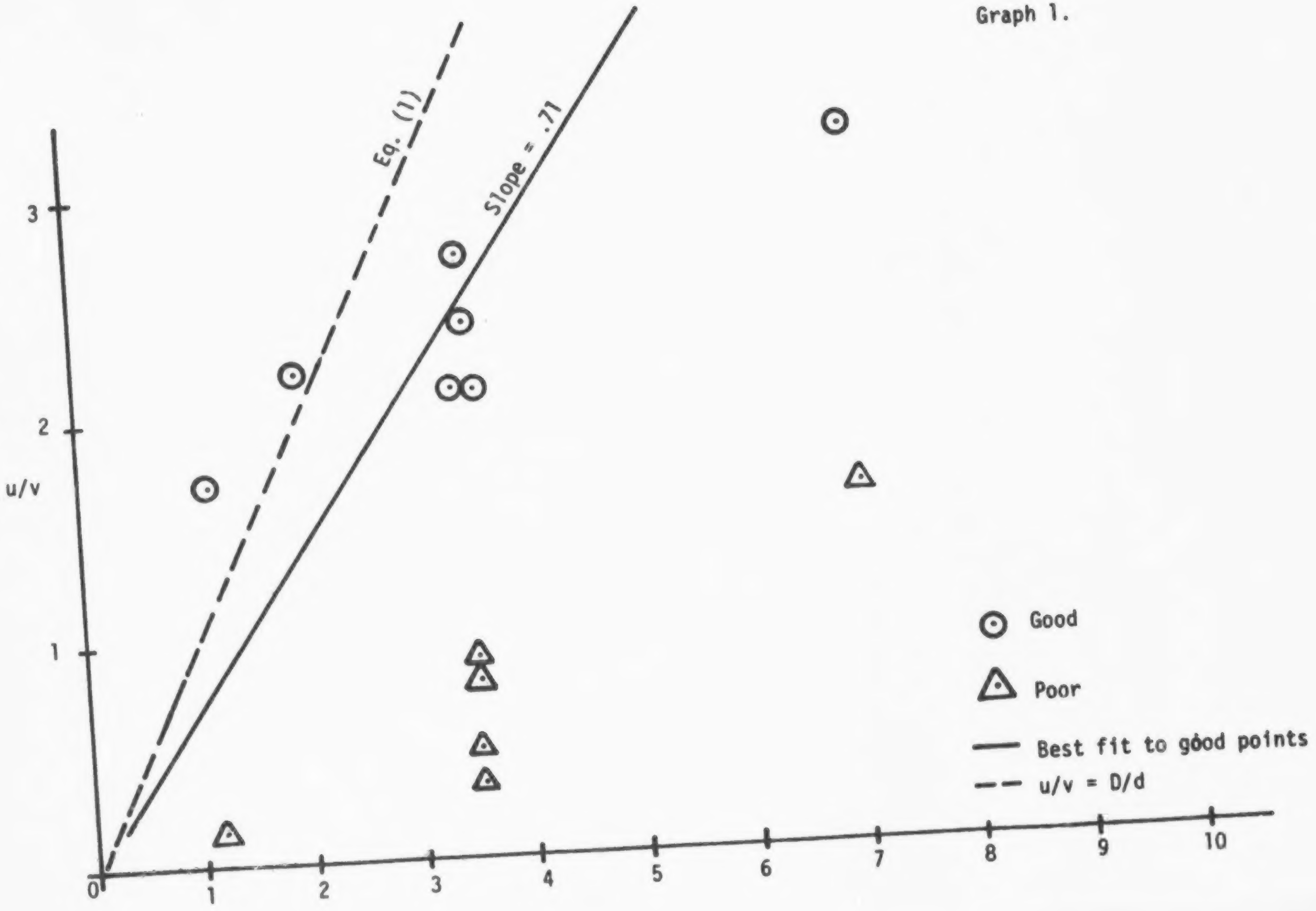
To check on the validity of Equation (1), we can look at the data of Chilton and Genereaux [47] in which mixing is defined as "good" if it occurs within 2 or 3 duct diameters from the orifice and "poor" if it occurs further downstream. This is shown on graph 1 in which u/v is plotted versus D/d for both good and poor runs. Also shown is the best fit curve to the "good" points. As it can be seen, Equation (1) holds reasonably well for the "good" points and that it does not fall near the "poor" runs.

It is possible to define the ratio of the mass flow in the duct to mass flow out of the orifice

$$MF = \frac{dm/dt\ (\text{orifice})}{dm/dt\ (\text{duct})} = \left(\frac{d}{D}\right)^2 \left(\frac{u}{V}\right)$$

and the ratio of kinetic energies per unit time in the duct and jet

Graph 1.


Eq. (1)
Slope = .71
u/v
3
2
1
0
1
2
3
4
5
6
7
8
9
10
Good
Poor
Best fit to good points
u/v = D/d

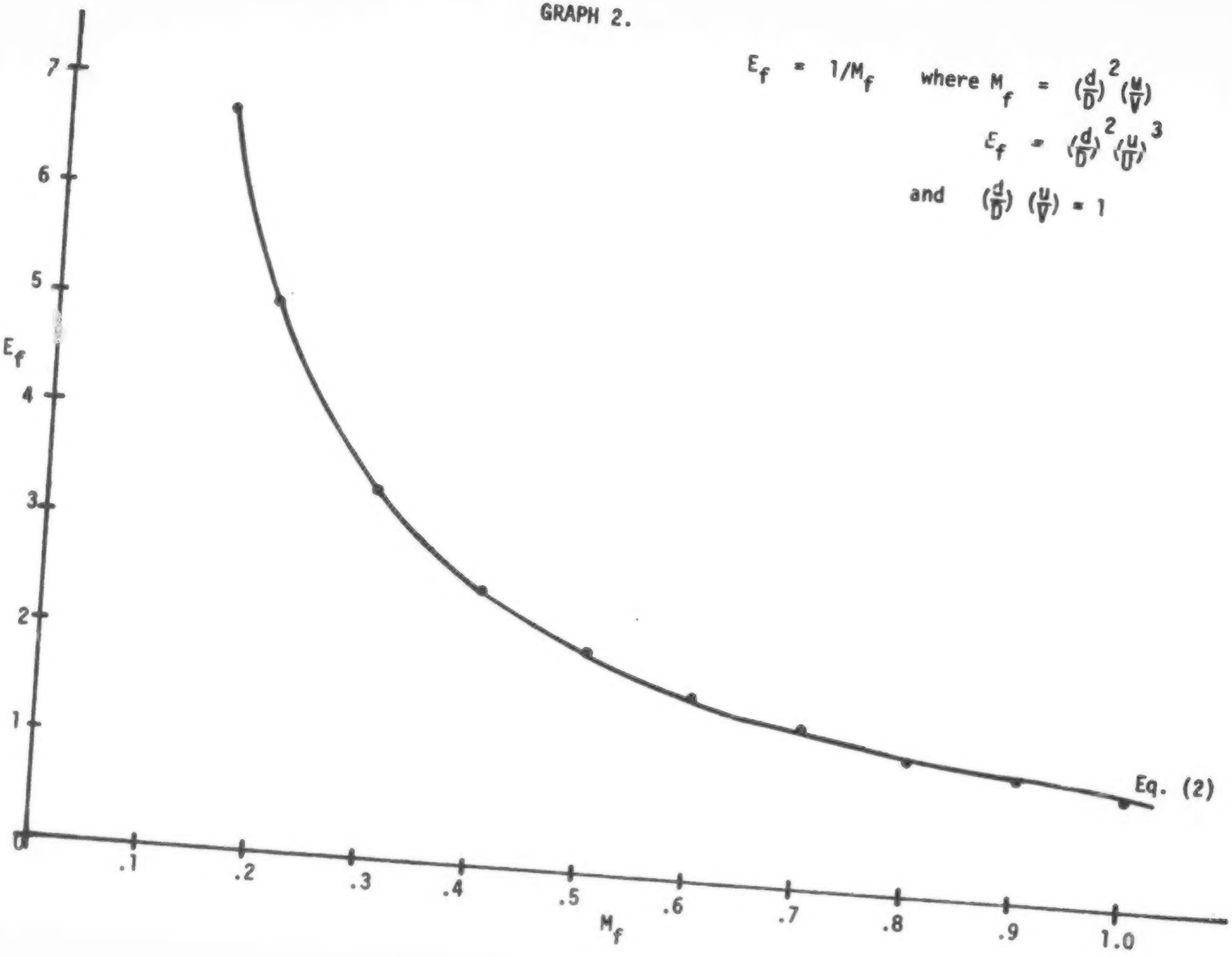
$$Ef = \frac{dE/dt}{dE/dt} \frac{(orifice)}{(duct)} = \left(\frac{d}{D}\right)^2 \quad \left(\frac{u}{V}\right)^3$$

Thus since $MfEf = \left(\frac{d}{D}\right)^4 \left(\frac{u}{V}\right)^4 = 1$ from equation (1) then

$$Ef = 1/Mf \qquad (2)$$

where $Mf < 1$. This is plotted on graph 2.

GRAPH 2.



$E_f = 1/M_f$ where $M_f = (\frac{d}{D})^2 (\frac{u}{V})$

$E_f = (\frac{d}{D})^2 (\frac{u}{U})^3$

and $(\frac{d}{D}) (\frac{u}{V}) = 1$

APPENDIX I

COMBUSTION AND MATERIAL BALANCE CALCULATIONS

Residual Fuel Oil Analysis*

Total Elemental Carbon:	85.36%
Total Elemental Hydrogen	11.45%
Total Elemental Sulfur	2.10%

Mystic Station on 4/24/74

	High Sulphur Fuel Oil Reading		
Time	Tank Gage	Volume (m^3)	
12:40 pm	22-10-1/2 α 120°C	5256.2	(33061.42 bbls)
2:40 pm	22-6-5/8 α 120°C	5181.8	(32593.78 bbls)
fuel usage =		74.4	(467.64 bbls)

.. fuel usage in kg/s assuming a specific gravity of oil = 0.96

$$\frac{74.4 \times 0.95 \times 1 \times 10^3}{3600 \times 2} = 9.92 \text{ kg/s}$$

Expected Emission

Assuming 3% of the sulfur goes to SO_3

$$SO_2(\text{kg/s}) = \frac{9.92 \times 2.1 \times 0.97 \times 64}{100 \times 32} = 405 \times 10^{-3} \text{ kg/s}$$

$$CO_2\ (\text{kg/s}) = \frac{9.92 \times 85.36 \times 44}{100 \times 12} = 31.0 \text{ kg/s}$$

---

[*] Average of duplicate analysis, see laboratory tests reports

LUVAK INC.
TEL: 617-[illegible]

722 MAIN STREET
BOYLSTON,
MASSACHUSETTS 01505

Analytical Report No. 0-9423

REQUESTED BY Walden Research
Division of Abcor, Inc.
201 Vasser Street
Cambridge, Mass. 02193

ATTENTION Ms. A. Zakak

Invoice Date 5/2/74

Invoice Number	Customer's Order Number	Customer's Requisition Number	Date Received
VF 1525	15328W		4/30/74

DESCRIPTION

1 sample analyzed for sulfur content in duplicate.

RESULTS

SAMPLE IDENTIFICATION	SULFUR
P.O. 15328W	2.05%
DUPLICATE	2.11%

LUVAK INC.
By

CALCULATIONS A

Moisture Calculation	North Duct	South Duct
Average Dry bulb temperature (T_D), °C	62	55
Average Wet bulb temperature (T_W), °C	49	47
Relative Humidity (from psy to chart) (% R.H.)	54	70
Saturated vapor pressure at dry bulb temp. (V.P.) N/m^2	21.5×10^3	15.7×10^3
Barometric Pressure (B.P.) N/m^2	100.9×10^3	100.9×10^3
Static pressure at moisture measuring device (P_s) N/m^2	3.7×10^3	3.2×10^3
Absolute static pressure at moisture measuring device ($B-P_s=P_s'$) N/m^2	97.2×10^3	97.7×10^3
% Moisture by volume $\frac{V.P. \times R.H.}{P_s'}$	$\frac{21.5 \times 50}{97.2}$	$\frac{15.7 \times 63}{97.7}$
% m	11	10

LABORATORIES AT KENILWORTH, N. J.
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LABORATORY ANALYSIS REPORT

SAMPLE DESIGNATED BY CLIENT AS No. 6 Fuel Oil

So. Boston, Mass.

May 2, 1974

Lab. No. 21318

(4) 755

FROM Submitted - 4/29/74
Walden Research Corp.
Cambr[illegible], Mass.

FOR Walden Research Corp.
201 Vassar Street
Cambridge, Mass. 02139

THIS LABORATORY REPORT MAY NOT BE PUBLISHED OR USED EXCEPT IN FULL. SHALL NOT BE USED FOR ADVERTISING OR IN CONNECTION WITH ADVERTISING OF ANY KIND UNLESS PERMISSION FOR THE PUBLISHING OR ADVERTISING OR AN APPROVED ABSTRACT HAS BEEN OBTAINED, IN WRITING FROM E. W. SAYBOLT & CO., INC. RESULTS ARE BASED ON ANALYSIS MADE AT THE TIME SAMPLES ARE RECEIVED AT LABORATORY.

SAMPLE	TESTS	RESULTS
#1	Total Elemental Carbon	85.39%
	Total Elemental Hydrogen	11.49%
#2	Total Elemental Carbon	85.34%
	Total Elemental Hydrogen	11.42%

E. W. SAYBOLT & CO., INC.

BY M.A. Fonseca

FORM 41 REV. 8/73

MEMBERS OF A. S. T. M. – A. P. I. – S. A. E.

SAMPLES, IF ANY, SHALL BE RETAINED FOR A PERIOD OF 45 DAYS UNLESS A LONGER PERIOD IS REQUESTED IN WRITING.

CALCULATIONS A

Moisture Calculation	North Duct	South Duct
Average Dry bulb temperature (T_D), °C	62	55
Average Wet bulb temperature (T_W), °C	49	47
Relative Humidity (from psy to chart) (% R.H.)	54	70
Saturated vapor pressure at dry bulb temp. (V.P.) N/m^2	21.5×10^3	15.7×10^3
Barometric Pressure (B.P.) N/m^2	100.9×10^3	100.9×10^3
Static pressure at moisture measuring device (P_s) N/m^2	3.7×10^3	3.2×10^3
Absolute static pressure at moisture measuring device ($B-P_s=P_s'$) N/m^2	97.2×10^3	97.7×10^3
% Moisture by volume $\frac{V.P. \times R.H.}{P_s'}$	$\frac{21.5 \times 50}{97.2}$	$\frac{15.7 \times 63}{97.7}$
% m	11	10

CALCULATIONS B

Molecular Weight and Relative Gas Density Calculation

$$M_a = \% CO_2 \times 44 + \% O_2 \times 32 + \frac{100 - (\% CO_2 + \% O_2)}{100} \times 28$$

$$M_s = Md\ (1 - \frac{\%M}{100}) + 18 \times \frac{\%M}{100}$$

$$G_d = \frac{M_s}{28.95}$$

	North Duct	South Duct
Average % CO_2	11.6	13.0
Average % O_2	6.3	5.2
% M	11	10
M_d	30.11	30.57
M_s	28.78	29.40
G_d	0.99	1.01

CALCULATIONS C

Emissions Calculations

Emission from North duct

Duct Area	$3.14 \times 3.35 = 10.52$, m^2
No. of Probes	9
% Moisture	11%
ΣSO_2 Concentration x velocity	121,000, ppm x m/s
ΣCO_2 Concentration x velocity	$1{,}530 \times 10^4$, ppm x m/s
Average duct temperature	411, °K
Average static pressure	97.2, $N/m^2 \times 10^3$

SO_2 Emission from North duct:

$$\frac{10.52}{9} \times \frac{121{,}000 \times 10^{-6} \times 64 \times 10^{-3}\ (1-0.11)}{\frac{101.33 \times 22.414 \times 411}{97.2 \times 273} \times 10^{-3}} = 229 \times 10^{-3}\ \text{kg/s}$$

CO_2 Emission from North duct:

$$\frac{10.52}{9} \times \frac{1{,}533 \times 10^{4} \times 10^{-6} \times 44 \times 10^{-3}\ (1-0.11)}{\frac{101.33 \times 22.414 \times 411}{97.2 \times 273} \times 10^{-3}} = 19.9\ \text{kg/s}$$

Emission from South duct

Duct Area	$3.14 \times 3.35 = 10.52\ m^2$
No. of probes	9
% Moisture	10%
ΣSO_2 Concentration x velocity	125,100, ppm x m/s
ΣCO_2 Concentration x velocity	$1{,}638 \times 10^4$, ppm x m/s
Average duct temperature	403, °K
Average static pressure	97.6, $N/m^2 \times 10^3$

SO_2 Emission from South duct:

$$\frac{10.52}{9} \times \frac{125{,}300 \times 10^{-6} \times 64 \times 10^{-3} \times (1-0.1)}{\frac{101.33 \times 22.414 \times 403}{97.8 \times 273} \times 10^{-3}} = 245 \times 10^{-3} \text{ kg/s}$$

CO_2 Emission from South duct

$$\frac{10.52}{9} \times \frac{1{,}637 \times 10^4 \times 10^{-6} \times 44 \times 10^{-3}\ (1-0.1)}{\frac{101.33}{97.8} \times \frac{22.414}{273} \times 403 \times 10^{-3}} = 22.1 \text{ kg/s}$$

Total SO_2 Emission from both South and North ducts is equal to

$$229 \times 10^{-3} + 246 \times 10^{-3} = 475 \times 10^{-3} \text{ kg/s}$$

% Error $\frac{475 - 405}{405} \times 100 = + 17\%$

Total CO_2 Emissions from both South and North ducts is equal to

22.1 kg/s + 19.9 kg/s = 42 kg/s

% Error $\frac{42 - 31}{31} = + 35\%$

In the following calculations (D and E), analysis of the system showed + 26% error on the measured flow rate, + 4% error on CO_2 analysis, and - 9% error on SO_2 analysis.

Total flow through North and South duct (Dry Basis)

$$\frac{10.52 \times 15 \times 273 \times 97.2 \times (1-0.11)}{411 \times 101.33} + \frac{10.52 \times 14 \times 273 \times 97.6\ (1-0.1)}{403 \times 101.33} =$$

$$89.48 + 86.49 = 175.97$$

$$= 176\ m^3/s \text{ at } 273°K \text{ and } 101.33\ N/m^2 \times 10^3$$

This is equivalent to 373,000 SCFM (Dry Basis).

---

* After the end of test and analysis of results the span gas of the Beckman instrument was checked against the Air Orsat analysis and was found to be about 4% lower than expected from an analysis made by the manufacturer. (span gas was believed to be10%; was found 9.6%).

CALCULATIONS D

Theoretical Combustion Calculations

Given Residual Fuel Oil

Carbon	85.36%
Hydrogen	11.45%
Sulfur	2.10%
Water Ash	negligible

Assume 18% excess Air

Theoretical combustion air requirement, 1 kg residual fuel oil basis
Standard Conditions

Carbon $C + O_2 \rightarrow CO_2$

$$0.8536 \times \frac{32}{12} = 2.276 \text{ kg } O_2 \equiv 22.414 \times 10^{-3} \times \frac{2.276 \times 10^3}{32} = 1.594 \text{ m}^3$$

or 7.59 m^3 of Air

Hydrogen $H_2 + 1/2\ O_2 \rightarrow H_2O$

$$0.1145 \times \frac{16}{2} = 0.916 \text{ kg } O_2 \equiv 22.414 \times 10^{-3} \times \frac{0.916 \times 10^3}{32} = 0.642 \text{ m}^3$$

or 3.06 m^3 of Air

Sulfur $S + O_2 \rightarrow SO_2$

$$0.021 \times \frac{32}{32} = 0.021 \text{ kg } O_2 \equiv 22.414 \times 10^{-3} \times \frac{0.021 \times 10^3}{32} = 0.0147 \text{ m}^3$$

or 0.070 m^3 of Air

Total Dry Air* Requirement

$$7.59 + 3.06 + 0.07 = 10.72\ m^3/kg\ of\ oil$$

$$10.72 \times 1.18 = 12.65\ m^3/kg\ of\ oil$$

Products of Combustion** at Standard Conditions/kg residual fuel oil

Carbon Dioxide

$$0.8536 \times 10^3\ \frac{44}{12} \times \frac{22.414 \times 10^{-3}}{44} = 1.594\ m^3$$

Water from combustion

$$0.1145 \times 10^3 \times \frac{18}{2} \times \frac{22.414 \times 10^{-3}}{18} = 1.283\ m^3$$

Nitrogen

$$12.65 \times 0.79 = 9.99\ m^3$$

Sulfur Oixdes as Sulfur Dioxide

$$0.021 \times 10^3 \times \frac{64}{12} \times \frac{22.414 \times 10^{-3}}{64} = 0.015\ m^3$$

Oxygen

$$12.65 \times (1.18 - 1) \times 0.21 = 0.48\ m^3$$

Total Dry Volume

$$1.594 + 9.99 + 0.015 + 0.48 = 12.08\ m^3/kg\ fuel\ oil$$

---

* Tambiant = 44°F, R.H. = 70

** Water in fuel or air is not accounted for

Estimated sulfur dioxide concentration (assume 3% goes to SO_3)

$$\frac{0.021 \times 10^3 \times 22.414 \times 10^{-3} \times 10^6 \times 0.97}{32 \times 12.08} = 1180 \text{ ppm (Dry Basis)}$$

Estimated Carbon dioxide concentration (neglect CO conversion)

$$\frac{0.8536 \times 10^3 \times 22.414 \times 10^{-3} \times 10^6}{12 \times 12.08} = 132{,}000 \text{ ppm or } 13.2\% \text{ (Dry Basis)}$$

Expected % moisture is near 10%. Slightly higher results were obtained (10% for South duct and 11% for North duct) because of leakage of the feed water at the economizer section during the test period.

CALCULATIONS E

Analysis of Results (Based on 18% excess air*)

Excess dilution due to air in leakage at the air preheaters (using 5.8% O_2 from Fyrites measurements):

$$\frac{x + 0.48}{12.08 + \frac{x}{0.21}} = 0.058$$

$$x + 0.48 = (12.08 + 4.76x)\ 0.058$$

$$x + 0.48 = 0.7 + 0.276\ x$$

$$0.724\ x = 0.22$$

$$x = 0.3$$

Expected SO_2 concentration after dilution $\dfrac{1180 \times 12.08}{\underset{(13.51)}{12.08 + 4.76 \times 0.3}}$ ~1055 ppm

Average SO_2 reading is equal to 955, i.e., 9% lower

Expected CO_2 concentration after dilution $\dfrac{13.2 \times 12.08}{12.08 + 4.76 \times 0.3}$ ~ 11.8%

Average CO_2 reading is equal to 12.3%, i.e., 4% higher.

---

* From Boston Edison personal communication about ~18% excess air was used during the test period

	All Standard Conditions (Dry Basis)
Expected dry flue gas volumetric rate 12.08 x 9.92 =	119.8 m^3/s (254,000 SCFM*)
Measured flow rate	176.0 m^3/s (373,000 SCFM**)
If +17% -)-9%) = +26% error is from flow measurement then the actual flow rate is equal to $\frac{176 - x}{x} = 0.26$	139.7 m^3/s* (296,000 SCFM)
Air in leakage is equal to 139.7 - 119.8 =	19.9 m^3/s (42,000 SCFM)
Oxygen in air in leakage 19.9 x 0.21 =	4.2 m^3/s
Theoretical oxygen from combustion 0.48 x 9.92 =	4.8 m^3/s
Total oxygen at the after air preheater section 4.2 + 4.8 =	9 m^3/s
% oxygen based on dry volume $\frac{9}{139.7} \times 100$ =	~6.4%
Average Measured % oxygen	5.8%
% difference	9%

---

* Actual Flow rate at ~ 407°K and 97.4 N/m^2 is equal to ~455,000 ACFM (DRY)

** Measured flow rate ~ 573,000 ACFM (DRY)

APPENDIX J.

COMPUTER PROGRAM FOR ANALYTICAL SIMULATION OF PROCEDURES FOR VELOCITY AND EMISSION MEASUREMENTS IN STRATIFIED STACK GASES

(THEORY AND USER MANUAL)

I. INTRODUCTION

A. PROGRAM FUNCTION

This program was developed to determine a measure of the error associated with the number, location, and area proportion assignment (strategy) for probes measuring effluent concentration and velocity in circular or rectangular flue gas ducts. It has been used to generate guidelines for selecting the number and best location of probes in ducts and the best means of weighting probe measurements (strategy) so as to determine the total and species effluent rate in a duct.

The program is designed such that the user specifies the duct geometry, actual concentration and velocity profiles in the duct and the number of probes and location and area associated with each probe. The program will compute the actual effluent rates:

$$\frac{\int_A CVdA}{\int_A dA} \quad \text{and} \quad \frac{\int_A VdA}{\int_A dA}$$

based upon the specified profiles and the rates that would be measured using the specified number, location, and calculation strategy of the probe placement. The error in the measured and actual effluent rate is then calculated as:

$$E_1 = \frac{\left| \int_A CVdA - \sum_i C_i V_i A_i \right|}{\int_A CVdA}$$

$$E_2 = \frac{\left| \int_A VdA - \sum_i V_i A_i \right|}{\int VdA}$$

By running a series of such cases, graphs may be produced showing the error in stack gas measurement (i.e., average velocity, effluent emission rate) as a function of number of probes, probe location, strategy, and concentration and velocity profile specification. These graphs may then be used as guidelines to determine the best number, location, and strategy for placing probes in a specific duct if some estimate for measurement (survey) of the duct profiles is available.

B. REPORT ARRANGEMENT

The main body of this report is made up of six primary sections as follows:

1. Computational Procedure, which briefly describes the analytical procedures through which the program operates.

2. Input Data Description, which contains a detailed description of the input data format for the program.

3. Output Data Description, which contains a brief description of the normal program output and the error messages generated by the program.

4. Sample Cases, which demonstrate the execution of the program.

5. Miscellaneous Operational Information, consisting of program capacity and modification procedures and of program and JCL requirements.

6. Exhibits, describing the detailed computational procedures of each subroutine and a source listing are included in the last section.

II. COMPUTATION PROCEDURE

There are seven general steps in the program's analysis procedure. In step one, the duct characteristics are read (shape and size) and the form of the concentration, velocity data are specified (raw data or polynomials). Step two deals with reading and editing raw concentration and velocity data and with generating least square polynomial curve fit coefficients for this data. Step three involves reading the order and coefficients of concentration and velocity data specified via polynomials. Step four involves printing and punching of polynomial coefficients for raw velocity and concentration data calculation and printing of given versus predicted velocity and concentration data (for velocity and concentration profiles specified as raw data) and of the standard deviation in concentration and velocity for these fits. Step five consists of the analytical integration of the velocity and concentration polynomials to predict "actual" average emissions, velocity, and concentration. In step six, the program reads, counts and edits the specified probe location data. Finally, in step seven, the program determines and prints the average concentration, velocity and emissions associated with the probe placement specified in step six, and determines the error associated with that simulated measurement procedure.

The details of these seven steps are outlined in greater detail, below.

Step 1. Determine duct characteristics and form of concentration velocity data by reading the first control card:

Geometry Code (NGC): 0 circular cross sectioned duct
1 rectangular cross sectioned duct
>1 end of Program (call EXIT)

Dimensions of the Duct: Radius for circular (A).
(A,B) Length and width for rectangular

Profile Data Code (NIC): 1 concentration and velocity profile data are given as the coefficients of the polynomial

$$Z=\sum_{i=1}^{N}\left(\sum_{j=1}^{M} a_{ji}\,YORT^{j-1}\right)XORR^{i-1}$$

YORT: $\left(\frac{y}{(B/2)}\right.$ or Theta, $\left.\frac{\theta}{2\pi}\right)$

XORR: $\left(\frac{X}{A/2}\right.$ or $\left.\left(\frac{R}{A}\right)\right)$

GO TO STEP 3

: 0 concentration and velocity raw data are given

GO TO STEP 2

Step 2. Read, edit and fit raw concentration and velocity data

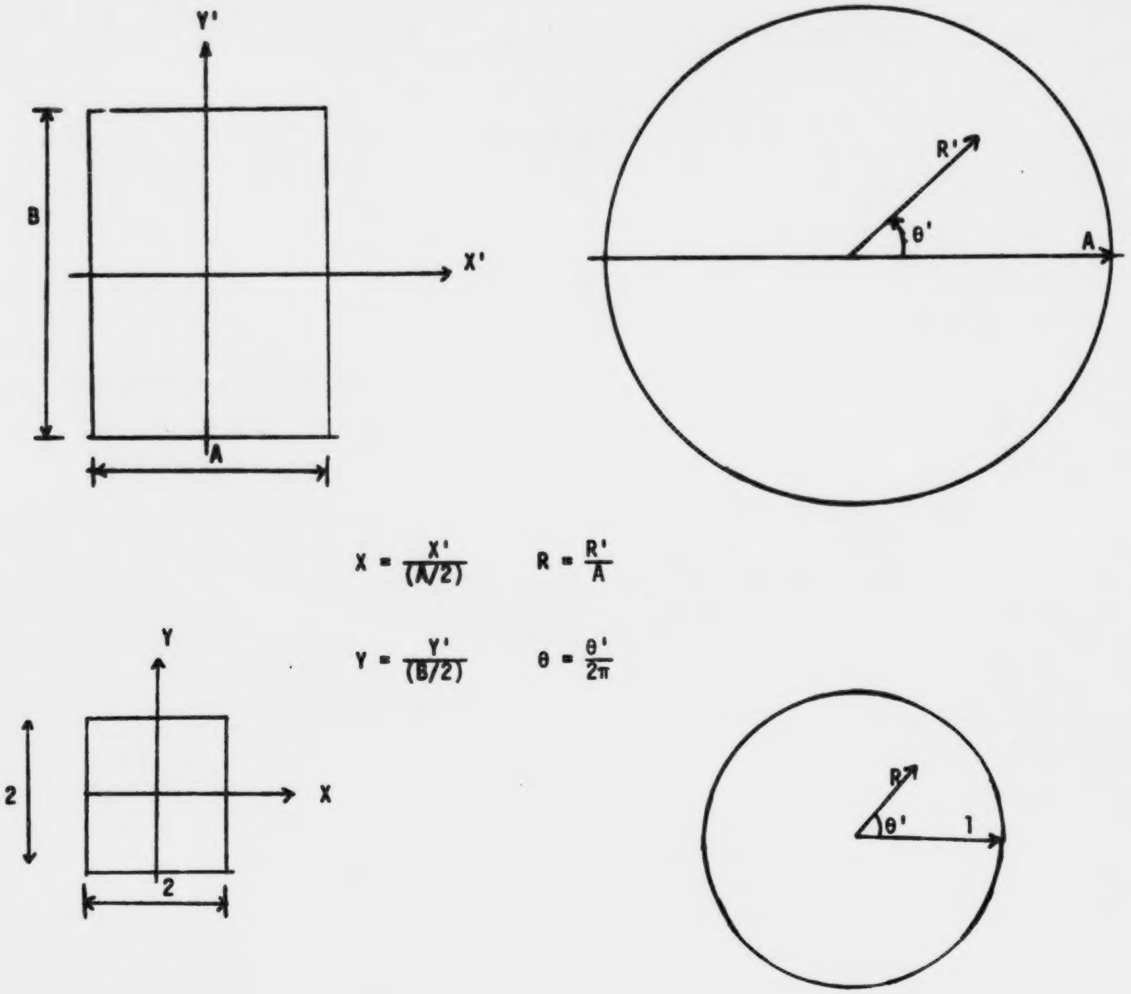
a. READ, COUNT, and EDIT concentration and velocity raw data. (A blank card with 9999 on column 77-80 after the last concentration or velocity data will provide a method to determine NCP, number of concentration data points, and NVP, number of velocity data points).

EDITING CRITERIA: concentration ≥ 0

for rectangular duct	$\lvert X' \rvert \leq \frac{A}{2}$ $\lvert Y' \rvert \leq \frac{B}{2}$
for curcular duct	$0 \leq R' \leq A$ $0 \leq \theta' \leq 2\pi$

Note: If an error is identified in the data, the program prints an error message, and continues to read and edit the remaining data. However, the program will default on analysis of the duct and associated probe assignments.

b. Normalize (X', Y') or (R', θ') to non-dimensional variables (X,Y) or (R,θ).


Y'
B
X'
A
R'
θ'
A
Y
2
X
2
R
θ'
1

$$X = \frac{X'}{(A/2)} \qquad R = \frac{R'}{A}$$

$$Y = \frac{Y'}{(B/2)} \qquad \theta = \frac{\theta'}{2\pi}$$

c. Read the number of maximum and minimum points in concentration and velocity input data, with respect to (X,Y) or R, θ. (MCXOR, MCYOT, MVXOR, MVYOT) (See note 5, input data).

d. Determine the order of the polynomials [{V(X,Y), C(X,Y)}, {V(R,Θ), C(R,Θ)}] to be generated.

Order of C(X,Y) or C(R,Θ) in X or R = NC = 2*MCXOR + 3
Order of C(X,Y) or C(R,Θ) in Y or Θ = MC = 2*MCYOT + 3
Order of V(X,Y) or V(R,Θ) in X or R = NV = 2*MVXOR + 3
Order of V(X,Y) or V(R,Θ) in Y or Θ = MV = 2*MVYOT + 3

If NC * MC>NCP; NC = $\sqrt{NCP}$, MC = $\frac{NCP}{NC}$

If NV * MV>NVP; NV = $\sqrt{NVP}$, MV = $\frac{NVP}{NV}$

e. Call Subroutine COEFG and CROUT (See Exhibits D and E) to find the coefficients of the least square polynomials for C and V. (Note that output arrays of these routines are printed as additional information). GO TO STEP 4.

Step 3. Read the order and coefficients of concentration and velocity polynomials.
GO TO STEP 4

Step 4. Print and punch curve fit data.

a. PRINT the coefficients of the polynomials. If NIC = 1, GO TO STEP 5, otherwise, continue as follows

b. PUNCH the coefficients of the polynomials, and PRINT the actual (given) and predicted (computed) velocity and concentration for each input data point.

c. Calculate and PRINT the standard deviation in C and V

$$\text{S.D. in C} = \sqrt{\frac{\Sigma(\text{computed C - actual C})^2}{\text{NCP}}}$$

$$\text{S.D. in V} = \sqrt{\frac{\Sigma(\text{computed V - actual V})^2}{\text{NVP}}}$$

Step 5. Evaluate average C, V, and CV. (based on polynomials)

$$\text{CI} = \frac{\int_A \text{CdA}}{\int_A \text{dA}}$$

$$\text{VI} = \frac{\int_A \text{VdA}}{\int_A \text{dA}}$$

$$\text{CVI} = \frac{\int_A \text{CVdA}}{\int_A \text{dA}}$$

Step 6. Read, count and edit the probe location data by reading the probe assignment control card.

If NST = 1: READ, COUNT, and EDIT, probe assignment data.

a. READ:

NASEG: (Number of equal area segments).
XORR(I), YORT(I), IARSEG(I) (Area segment number).

b. COUNT: The blank card with 9999 on column 77-80 after the last probe data card indicates number of probes (NPR)

c. EDIT: for rectangular ducts: $|XORR'| \leq \frac{A}{2}$, $|YORT'| \leq \frac{B}{2}$

for circular ducts: $o \leq XORR' \leq A$, $o \leq YORT' \leq 2\pi$

for all ducts: $o < IARSEG \leq NASEG$

If any error is uncovered in these steps, the program prints an error message, ignores this probe assignment strategy, and returns to Step 6, to read the next probe assignment control card.

If no error, XORR', YORT' are made dimensionless by:

rectangular ducts: $XORR = \frac{XORR'}{A/2}$ $YORT = \frac{YORT'}{B/2}$

circular ducts: $XORR = \frac{XORR'}{A}$ $YORT = \frac{YORT'}{2\pi}$

GO TO STEP 7.

If NST = o (new duct) then GO TO STEP 1 for a new duct

If NST >1 CALL EXIT (also reserved for future modification of program)

Step 7. Determine average concentration, emission, and velocity for the given strategy.

a. PRINT the probe locations and probe area segment number.

b. CALL Subroutine PRBANS (see Exhibit G) to find average concentration (CBAR), velocity (VBAR), and emission (CVBAR) based on the EAUAL AREA STRATEGY.

c. Find the errors: $E1 = \left|1. - \frac{CVBAR}{CVI}\right|$

$$E2 = \left|1. - \frac{CBAR*VBAR}{CVI}\right|$$

$$E3 = \left|1. - \frac{VBAR}{VI}\right|$$

d. PRINT actual (VI, CVI) and predicted (VBAR, CVBAR, CBAR) average velocities, emissions, and concentrations.

e. PRINT, E1, E2, and E3

GO TO STEP 6 to read the next probe assignment control card.

III. INPUT DESCRIPTION

A. GENERAL DESCRIPTION

Input data to the program are specified by use of thirteen basic card formats specified in Figure 1. Card types one - two are used for specification of duct geometry and concentration-velocity data format. Card types three - nine are used for specification of the concentration and velocity profiles. Specifically, card types three - six are employed if the profiles are specified as raw data; types seven - nine are for use with data specified via polynomials. Card types ten - thirteen are used for specification of sampling strategy, and probe location.

Card type one is used for inputting duct shape (rectangular or circular), duct dimensions (X, Y or R), and input velocity-concentration data format (raw data or polynomial coefficients). Card type two is a single comment card employed by the user for identification of the duct.

Card type three is used to specify concentration and geometric coordinate data (X, Y or R,Θ) for raw concentration data. Card type four is an indicator card used to signify the end of the concentration input data. Card type five is for use with actual velocity data, whereon the user specifies velocity and geometric coordinate data (X, Y or R,Θ). Card type four is then reused as an indicator to signify the end of the input velocity data. Card type six follows the raw concentration and velocity data, specifying the number of maxima and minima in planes of constant Y(or Θ) and X (or R) in the concentration and velocity profiles.

If the user specified concentration and velocity profiles via polynomials, card types three - six are omitted and card types seven - nine employed. On card type seven, the user specifies the order + 1 of the concentration and velocity polynomials as functions of X(or R) and Y(or Θ). In card types eight and nine the concentration and velocity polynomial coefficient are respectively specified.

Figure 1. INPUT DATA FORM

Card Type	Number		Card Column															
1	1 Only	NGC, GEOMETRY CODE: (rectangular,1)&(Circular,0)	1															
		A, (Note 1)	3	4	5	6	7	8	9 .	10	11	12	13	14				
		B, (Note 2)	27	28	29	30	31	32	33 .	34	35	36	37	38				
		NIC, (Note 3)	50															
2	1 Only	Any comment for the duct	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
			17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
			33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
			49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64
			65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
3	None or less than 300	CONCENTRATION	1	2	3	4	5	6	7 .	8	9	10	11	12				
		X or R Location $\left(-\frac{A}{2}\le X\le\frac{A}{2}\right)$, $(0\le R\le A)$	13	14	15	16	17	18	19 .	20	21	22	23	24				
		Y or θ Location $\left(-\frac{B}{2}\le Y\le\frac{B}{2}\right)$, $(0\le\theta\le 2\pi)$	25	26	27	28	29	30	31 .	32	33	34	35	36				
4	1 Only	Last card indicator (Note 4)	77 9	78 9	79 9	80 9												
5	None or less than 300	VELOCITY (Positive velocity toward stack outlet)	1	2	3	4	5	6	7 .	8	9	10	11	12				
		X or R location $\left(-\frac{A}{2}\le X\le\frac{A}{2}\right)$, $(0\le R\le A)$	13	14	15	16	17	18	19 .	20	21	22	23	24				
		Y or θ location $\left(-\frac{B}{2}\le Y\le\frac{B}{2}\right)$, $(0\le\theta\le 2\pi)$	25	26	27	28	29	30	31 .	32	33	34	35	36				

NOTES:

1) A is radius of circular duct or rectangular dimension in X direction.

2) B is 0 for circular duct and dimension in Y direction of Rectangular duct.

3) NIC = 0 Input data on profiles as raw data points, use card forms 3, 5, and 6 but not 7, 8, and 9.
= 1, Input data on profiles as polynomial coefficients, use card forms 7, 8, and 9 but not 3, 5, and 6.

4) PUT after the last card 3, last card 5, last card 13.

Figure 1. INPUT DATA FORM, Cont.

Card Type	Number		Card Column															
6	None or 1	MCXOR, (Note 5) ≤ 4	1	2														
		MCYOT, (Note 5) ≤ 4	3	4														
		MVXOR, (Note 5) ≤ 4	5	6														
		MVYOT, (Note 5) ≤ 4	7	8														
7	None or 1	NC, Order + 1 of polynomial C as a function of x or R	1	2														
		MC, Order + 1 of polynomial C as a function of Y or θ	3	4														
		NV, Order + 1 of polynomial V as a function of x or R	5	6														
		MV, Order + 1 of polynomial V as a function of Y or θ	7	8														
8	None or $\frac{NC*MC}{5}$	C(1,1) (See note 6)	1	2	3	4	5	6	7.	8	9	10	11	12	13	14	15	16
		C(2,1)	17	18	19	20	21	22	23.	24	25	26	27	28	29	30	31	32
		⋮	33	34	35	36	37	38	39.	40	41	42	43	44	45	46	47	48
		C(MC,1)	49	50	51	52	53	54	55.	56	57	58	59	60	61	62	63	64
		C(MC,NC)	65	66	67	68	69	70	71.	72	73	74	75	76	77	78	79	80
9	None or $\frac{NV*MV}{5}$	V(1,1) (Note 6)	1						7.									16
		V(2,1)	17						23.									32
		⋮	33						39.									48
		V(MC,1)	49						55.									64
		V(MC,NC)	65						71.									80

NOTES:

5) In order for the program to find the least squar concentration and velocity polynomials [(V(x,y), C(x,y)), (V(R,θ), C(R,θ))], the user must provide the order of these polynomials by specifying MCXOR, MCYOT, MVXOR, and MVYOT where:

1 + order of C(X,Y) or C(R,θ) in X or R = NC = 2*MCXOR + 3 1 + order of V(X,Y) or V(R,θ) in X or R = NV = 2*MVXOR + 3

1 + order of C(x,y) or C(R,θ) in Y or θ = MC = 2*MCYOT + 3 1 + order of V(X,Y) or V(R,θ) in Y or θ = MV = 2*MVYOT + 3

MCXOR, MCYOT, MVXOR, and MVYOT can be any digits between -1 and 4 (inclusive) based on the user's judgement of the profile shape. Lacking a suitable basis for selecting values, the following definitions lead to acceptable orders:

MCXOR: Number of maxima and minima in a plane of constant Y or θ in concentration profile
MCYOT: Number of maxima and minima in a plane of constant X or R in concentration profile
MVXOR: Number of maxima and minima in a plane of constant Y or θ in velocity profile
MVYOT: Number of maxima and minima in a plane of constant X or R in velocity profile

6) Coefficients of concentration and velocity polynomials must be in the order of (1,1), (2,1), ..., (MC,1), ..., (MC,NC).

$$\text{Concentration} = \sum_{i=1}^{NC} \sum_{j=1}^{MC} C(J,I) \left(\frac{2Y}{B}\right)^{J-1} \left(\frac{2X}{A}\right)^{I-1}, \qquad \text{Velocity} = \sum_{i=1}^{NV} \sum_{j=1}^{MV} V(J,I) \left(\frac{2Y}{B}\right)^{J-1} \left(\frac{2X}{A}\right)^{I-1}$$

Figure 1. INPUT DATA FORM, Cont.

Card Type	Number		Card Column															
10	1	NST, Strategy Code, (Note 7)	1															
11	1	Strategy Comment Card	1															16
			17															32
			33															48
			49															64
			65															80
12	1	NASEG, Number of equal area segments	1	2	3													
13	Less than 300	X or R probe location $-\frac{A}{2} \le X \le \frac{A}{2}$, $0 \le R \le A$	1	2	3	4	5	6	7 .	8	9	10	11	12				
		Y or θ probe location $-\frac{B}{2} \le Y \le \frac{B}{2}$, $0 \le \theta \le 2\pi$	13	14	15	16	17	18	19 .	20	21	22	23	24				
		IARSEG, Number of area segment in which probe is located	25	26	27													

NOTES:
7) NST = 0 For new duct
= 1 Equal Area Strategy; followed by card types 11, 12, 13, 4
= 2, 3, 4, and 5 is reserved for other strategies; (Call EXIT).

IMPORTANT NOTE: Dimensions of A, B, X, Y, R, concentration, and velocity must be consistent, e.g.,
A, B, X, Y, and R in feet
Concentration in lb or lb mole/ft^3
Velocity in ft/sec
θ in radian

$$C(j,i) = \frac{lb}{ft^3}, \quad V(j,i) = \frac{ft}{sec}$$

Card types ten - thirteen are used for specification of sample probe location. Card type ten is used as an indicator of the strategy type and to signify the end of the examination of a given duct and profile set. Card type eleven is a comment card for the user to specify the type of sampling strategy under evaluation. Card type twelve is used with equal area strategies and indicates the number of equal area segments involved in the sampling strategy. Card type thirteen is employed to specify each probe geometric location (X,Y, or R,Θ) and area segment assignment.

B. DETAILED DESCRIPTION OF INPUT DATA

Figure 1 clearly details the format of the input data. The user is advised to be assured of the following special considerations:

1. If raw data is specified for the concentration and velocity profiles, no more than 300 discrete concentration and 300 discrete velocity points may be entered.

2. The order of the velocity and concentration polynomials is specified as the order + 1. A maximum order of 10 (order + 1 = 11) is allowed, i.e., NC, NV, MC, MV $\leq$ 11.

3. For each probe assignment case, the maximum number of probes and equal area segments is 300.

4. Any set of self-consistent units are acceptable in the analysis procedure.

Figure 2 illustrates the input data structure for the program. The user may examine one duct and one strategy (Case 1), one duct and n strategies (Case 2) or m ducts and n strategies (Case 3). There are no internal limits on m (number of ducts) and n (number of probe assignment strategies in the program). The program is terminated by reading card type one with geometry code of 2, as shown on Figure 2.

INPUT DATA STRUCTURE

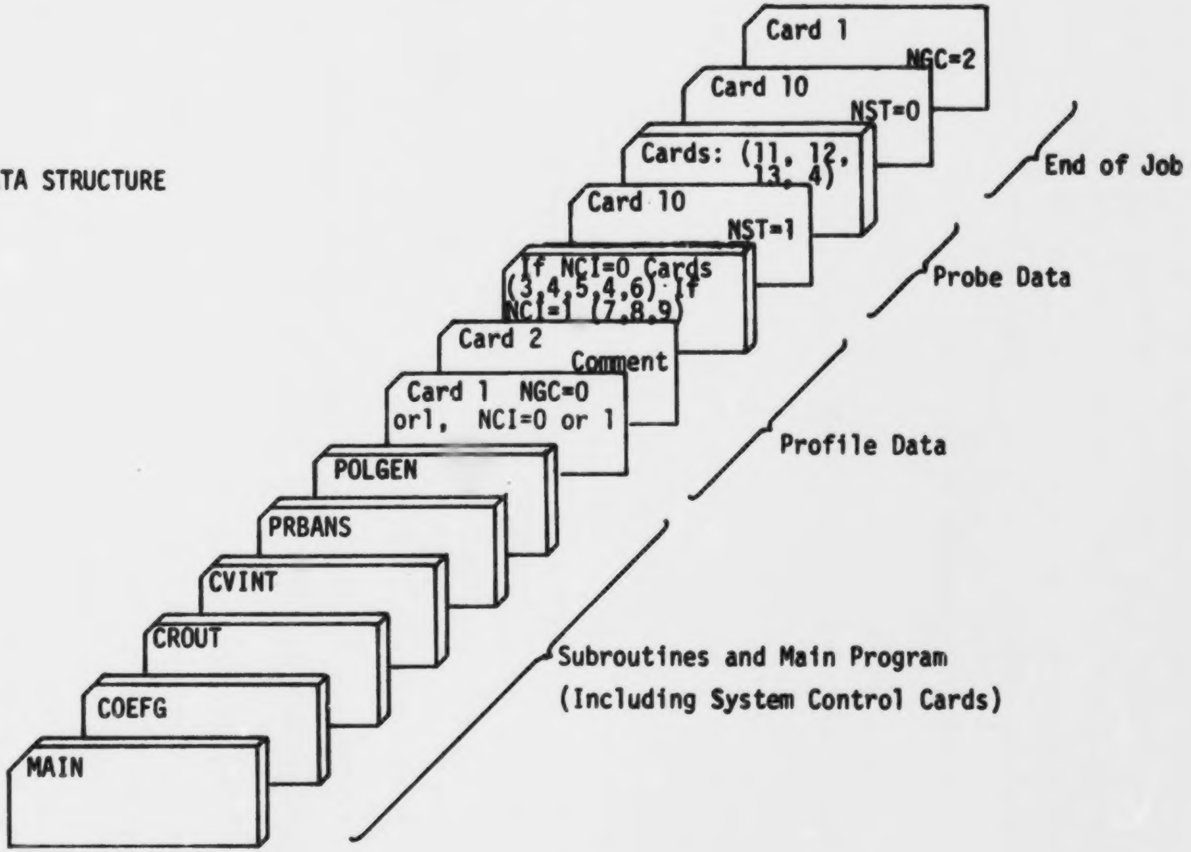

Card 1
NGC=2
Card 10
NST=0
Cards: (11, 12, 13, 4)
Card 10
NST=1
If NCI=0 Cards (3,4,5,4,6) If NCI=1 (7,8,9)
Card 2
Comment
Card 1 NGC=0 or1, NCI=0 or 1
POLGEN
PRBANS
CVINT
CROUT
COEFG
MAIN
End of Job
Probe Data
Profile Data
Subroutines and Main Program
(Including System Control Cards)

Figure 2. Case 1. One Duct and One Strategy Only

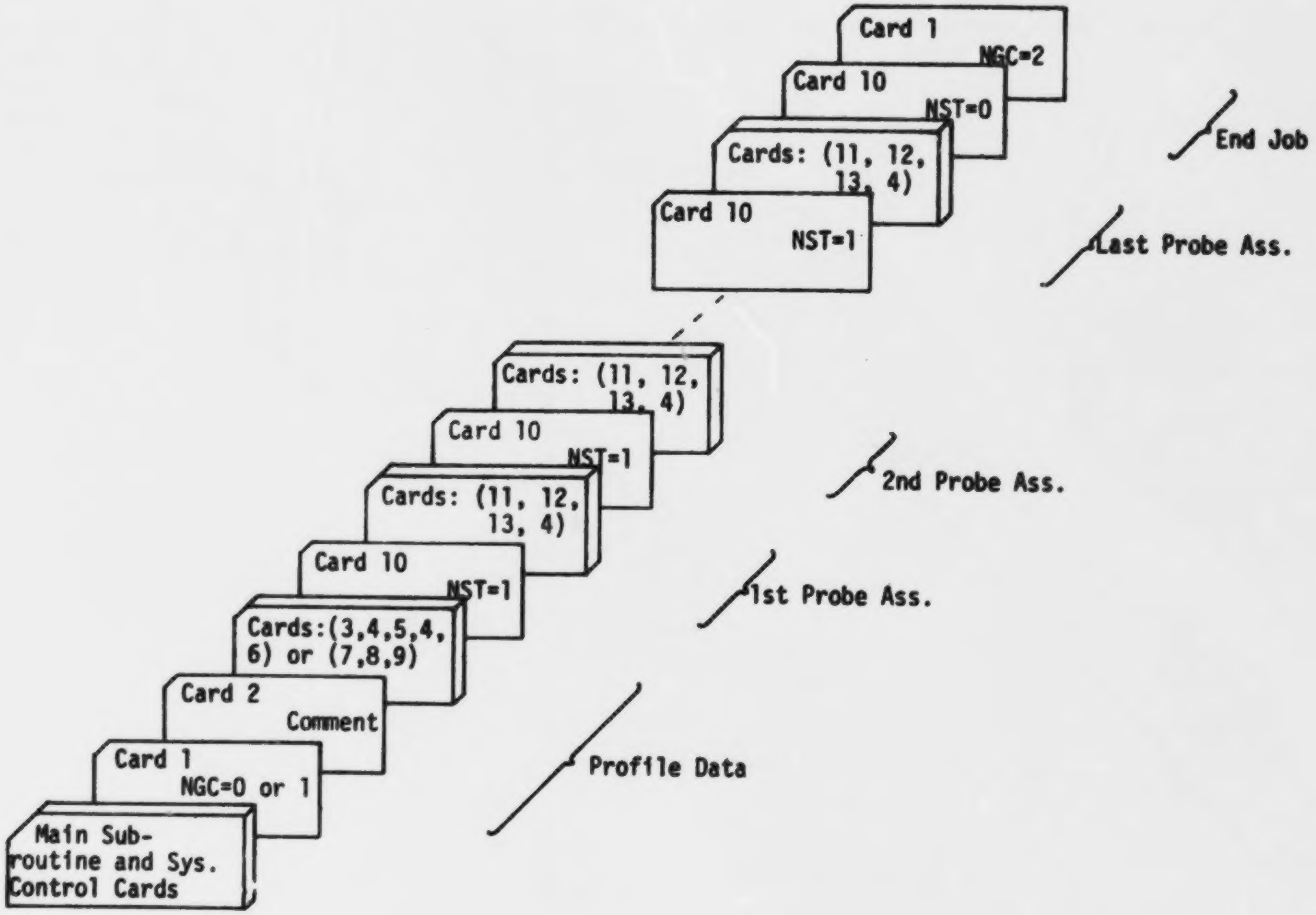

Card 1
NGC=2
Card 10
NST=0
Cards: (11, 12, 13, 4)
Card 10
NST=1
End Job
Last Probe Ass.
Cards: (11, 12, 13, 4)
Card 10
NST=1
2nd Probe Ass.
Cards: (11, 12, 13, 4)
Card 10
NST=1
1st Probe Ass.
Cards:(3,4,5,4, 6) or (7,8,9)
Card 2
Comment
Profile Data
Card 1
NGC=0 or 1
Main Subroutine and Sys. Control Cards

Figure 2. Case 2. One Duct Only With More Than One Strategy

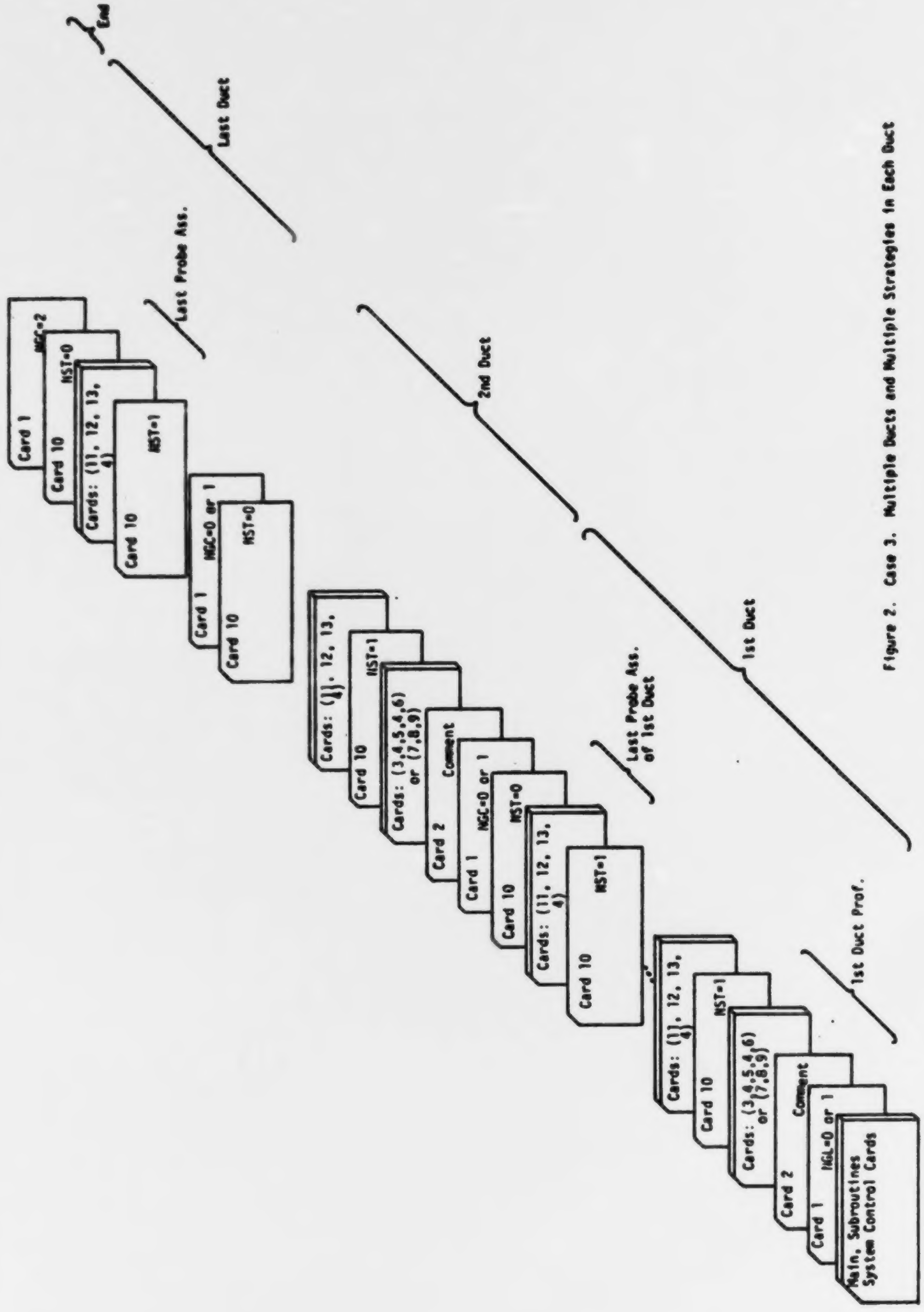

Main, Subroutines System Control Cards
Card 1
NGL=0 or 1
Card 2
Comment
Cards: (3,4,5,4,6) or (7,8,9)
Card 10
NST=1
Cards: (11, 12, 13, 4)
1st Duct Prof.
Card 10
NST=1
Cards: (11, 12, 13, 4)
Card 10
NST=0
Card 1
NGC=0 or 1
Card 2
Comment
Cards: (3,4,5,4,6) or (7,8,9)
Card 10
NST=1
Cards: (11, 12, 13, 4)
Last Probe Ass. of 1st Duct
1st Duct
Card 10
NST=0
Card 1
NGC=0 or 1
Card 10
NST=1
Cards: (11, 12, 13, 4)
Card 10
NST=0
Card 1
NGC=2
Last Probe Ass.
2nd Duct
Last Duct
End

Figure 2. Case 3. Multiple Ducts and Multiple Strategies in Each Duct

IV. OUTPUT DESCRIPTION

The output of the program consists of both printed and punched data. The data falls into two main categories: normal output and error messages. The normal output consists of the printed and punched results usually obtained with each run of the program. If any difficulties are encountered in the solution of a case, one or more error messages are printed. These messages are diagnostic statements which describe the nature of the difficulty. They are described in greater detail in sub-section IV-B.

A. NORMAL OUTPUT

Case 1: When the velocity and concentration profiles are provided as raw data, printed output consists of:

1. Number of input concentration and velocity data cards.
2. Input and output arrays from subroutine CROUT for concentration and velocity.
3. Stack geometry.
4. Concentration and velocity polynomials coefficients.
5. Results of profile correlation (actual versus predicted velocity and concentration values at input locations).
6. Probe location data for strategy.
7. Actual and predicted average velocity, average rate of emission, and average concentration.
8. Error of predicted average velocity and emission for the strategy.

Printed outputs 6, 7, and 8 are repeated for each specified strategy. Concentration and velocity polynomial coefficients (Item 4 above) are also punched for subsequent studies of the same duct - profiles.

Case 2: When the velocity and concentration profiles are specified through polynomial coefficients, there is no punched output and the printed output is the same as Case 1, except the above Items 1, 2, and 5 are omitted.

NOTE: For details, see output of sample cases.

B. ERROR MESSAGES

Five diagnostic error messages are printed by the program, each specific to an error condition which might occur in a given duct - strategy case. These conditions, the error message, and the program response are detailed below:

ERROR MESSAGE	ERROR CONDITION	PROGRAM RESPONSE
1. Error in entering concentration profile data (Card No. I)	CON(I)<0 for rectangular duct: $\|XORR(I)\| > \frac{A}{2}$, $\|YORT(I)\| > \frac{B}{2}$ for circular duct: XORR(I)<0, XORR(I)>A YORT(I)<0, YORT(I)>2π	The program edits the remaining data, but this duct is aborted. The next duct is evaluated, if available.
2. Error in entering velocity profile data (I)	for rectangular duct $\|XRV(I)\| > \frac{A}{2}$, $\|YTV(I)\| > \frac{B}{2}$ for circular duct XRV(I)<0 XRV(I)>A YTV(I)<0 YTV(I)>2π	Same as response immediately above.

ERROR MESSAGE	ERROR CONDITION	PROGRAM RESPONSE
3. Unable to solve by this method problem aborted.	CROUT routine unable to find the coefficients (division by zero or small numbers unavoidable).	Same as above response.
4. Probe (I) located outside duct boundaries. Problem aborted.	for rectangular duct $\lvert XORR'(I)\rvert > \frac{A}{2}$ $\lvert YORT'(I)\rvert > \frac{B}{2}$ for circular duct $XORR'(I)<0$, $XORR'(I)>A$ $YORT'(I)<0$, $YORT'(I)>2\pi$	The program edits the remaining data of this probe assignment strategy, but the analysis is aborted. The program executes the next case (if it exists).
5. Probe (I) assigned to a non existing area segment. Problem aborted.	IARSEG(I)<0 IARSEG(I)>NASEG	Same as response immediatly above.

V. SAMPLE CASES

This section consists of the following:

A. PROBLEM 1
  (a) Problem Statement
  (b) Data Preparation

B. PROBLEM 2
  (a) Problem Statement
  (b) Data Preparation

C. PROBLEM 3
  (a) Problem Statement
  (b) Data Preparation

D. SAMPLE CASES INPUT DATA

E. SAMPLE CASES OUTPUT DATA

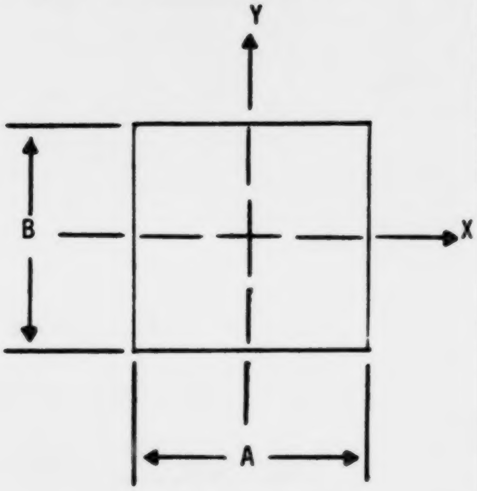
F. RESULTS AND CONCLUSIONS
  (a) Problem 1
  (b) Problem 2
  (c) Problem 3

A. PROBLEM 1

(a) Problem Statement

The concentration and velocity raw data for a duct of rectangular cross section is given as follows:

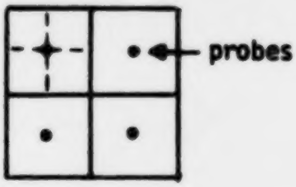
X	Y	V	C
-1.0	-1.0	0.0	0.0
-1.0	-0.5	0.0	0.0
-1.0	0.0	0.0	0.0
-1.0	+0.5	0.0	0.0
-1.0	+1.0	0.0	0.0
-0.5	-1.0	0.0	0.0
-0.5	-0.5	0.5625	0.5626
-0.5	0.0	0.7500	0.7500
-0.5	0.5	0.5625	0.5625
-0.5	1.0	0.0	0.0
0.0	-1.0	0.0	0.0
0.0	-0.5	0.7500	0.7500
0.0	0.0	1.0	1.0
0.0	0.5	0.7500	0.7500
0.0	1.0	0.0	0.0
0.5	-1.0	0.0	0.0
0.5	-0.5	0.5625	0.5625
0.5	0.0	0.7500	0.7500
0.5	0.5	0.5625	0.5625
0.5	1.0	0.0	0.0
1.0	-1.0	0.0	0.0
1.0	-0.5	0.0	0.0
1.0	0.0	0.0	0.0
1.0	0.5	0.0	0.0
1.0	1.0	0.0	0.0



A = B = 2 ft.

a.1. Find the concentration and velocity polynomials, and average C, V, and $\overline{CV}$ (emission).

a.2. What is the predicted average c, v, and cv, if 4 probes are located as shown.



a.3. Repeat a.2. for one probe in center.

(b) DATA PREPARATION

Card type	Data
(1)	NGC = 1 (rectangular) A = 2 B = 2 NIC = 0 (raw data)
(2)	Comment Sample problem one
(3)	Concentration raw data C X Y . . . .
(4)	 9999
(5)	Velocity raw data C X Y . . . .
(4)	9999

(6) Based on the raw data, we assumed that the concentration and velocity polynomials were second order in X and Y ∴ MCXOR=MCYOT=MVXOR=MVYOT=0

(10) Strategy Code
NST = 1

(11) Strategy Comment
Problem one, a.1 (4 probes)

(12) NASEG = 4

(13) Probe Locations

X	Y	Area Seg. No.
-.5	+.5	1
-.5	-.5	2
.5	.5	3
.5	-.5	4

(4) 9999

(10) Strategy Code
NST = 1

(11) Strategy Comment
Probelm one, a.2 (1 probe in center)

(12) NASEG = 1

(13) Probe Locations

X	Y	Area Seg. No.
0.	0.	1

(4) 9999

(10) Strategy Code
NST = 0 (End of problem 1)

B. PROBLEM 2

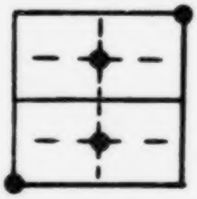
(a) Problem Statement

The concentration and velocity polynomials for above rectangular duct is given as:

$$C = \left[1 - \left(\frac{2X}{A}\right)^2\right] \left[1 - \left(\frac{2Y}{B}\right)^2\right] \qquad A = B = 2$$

$$V = \left[1 - \left(\frac{2X}{A}\right)^2\right] \left[1 - \left(\frac{2Y}{B}\right)^2\right]$$

a.1) What is the predicted average C, V, and (CV) if 4 probes are located as shown:



(b) Data Preparation

Card Type	Data
(1)	NGC = 1 (rectangular) A = 2 B = 2 NIC = 1 (polynomial coefficients given)
(2)	Sample Problem 2

Card Type	Data
(7)	NC = 3 MC = 3 NV = 3 MV = 3
(8)	Concentration polynomial is,

$$C = \left[1 + (0)\left(\frac{2Y}{B}\right) - \left(\frac{2Y}{B}\right)^2\right](1) + \left[1 + (0)\left(\frac{2Y}{B}\right) - \left(\frac{2Y}{B}\right)^2\right] *$$

$$(0)\left(\frac{2X}{A}\right) + \left[1 + (0)\left(\frac{2Y}{B}\right) - \left(\frac{2Y}{B}\right)^2\right] (-1)\left(\frac{2X}{A}\right)^2$$

$$C = \left[1 + (0)\left(\frac{2Y}{B}\right) - \left(\frac{2Y}{B}\right)^2\right] + \left[(0) + (0)\left(\frac{2Y}{B}\right) + (0)\left(\frac{2Y}{B}\right)^2\right] *$$

$$\left(\frac{2X}{A}\right) + \left[-1 + (0)\left(\frac{2Y}{B}\right) + \left(\frac{2Y}{B}\right)^2\right]\left(\frac{2X}{A}\right)^2$$

Therefore the concentration coefficients are as follows:

$C_{11} = 1.$ $C_{12} = 0.$ $C_{13} = -1.$
$C_{21} = 0.$ $C_{22} = 0.$ $C_{23} = 0.$
$C_{31} = -1.$ $C_{32} = 0.$ $C_{33} = 1.$

(9) Similarly, the velocity coefficients are:

$V_{11} = 1.$ $V_{12} = 0$ $V_{13} = -1.$
$V_{21} = 0.$ $V_{22} = 0$ $V_{23} = 0.$
$V_{31} = -1.$ $V_{23} = 0$ $V_{33} = 1.$

Card Type	Data
(10)	NST = 1
(11)	Problem 2, a.1 (4 probes)
(12)	NASEG = 2
(13)	Probe Locations

X	Y	Area Seg. No.
1.	1.	1
0.	.5	1
0.	- .5	2
-1.	-1.	2

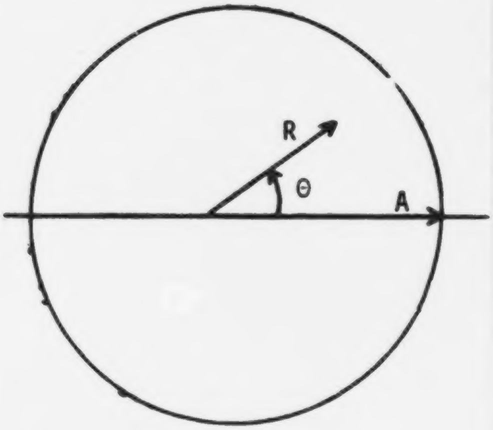
(4)	 9999
(10)	NST = 0 (End of Problem 2)

C. PROBLEM 3

(a) Problem Statement

The concentration and velocity raw data for a duct of circular cross section is given as follows:

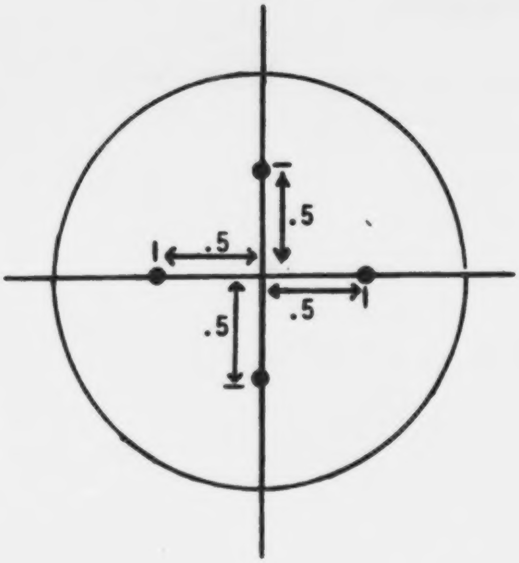
R	θ (Radians)	C
0.0	0.0	2.0
0.0	3.14159	2.0
0.25	0.0	2.0625
0.25	3.14159	2.0265
0.5	0.0	2.25
0.5	3.14159	2.25
0.75	0.0	2.5625
0.75	3.14159	2.5625
1.0	0.0	3.0
1.0	3.14159	3.0



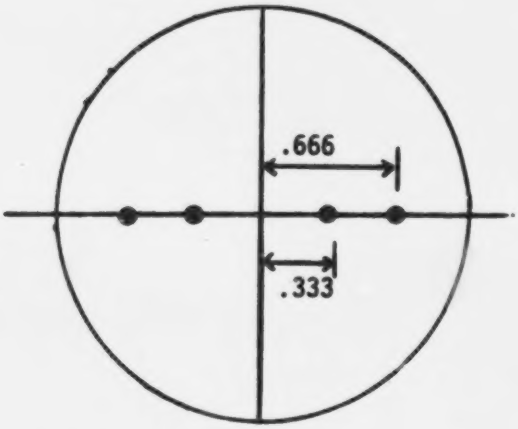
A = 1

R	θ	V
0.0	0.0	2250
0.0	1.57080	2250
0.0	3.14159	2250
0.0	4.71239	2250
0.0	6.28318	2250
0.25	0.0	1687.5
0.25	1.57080	1652.34375
0.25	3.14159	1640.625
0.25	4.71239	1652.34375
0.25	6.28318	1687.5
0.5	0.0	1125
0.5	1.57080	1078.125
0.5	3.14159	1062.5
0.5	4.71239	1078.125
0.5	6.28318	1125
0.75	0.0	562.5
0.75	1.57080	527.34375
0.75	3.14159	515.625
0.75	4.71239	527.34375
0.75	6.28318	562.5
1.0	0.0	0.0
1.0	1.57080	0.0
1.0	3.14159	0.0
1.0	4.71239	0.0
1.0	6.28318	0.0

a.1) Find the predicted average c, v, and $(\overline{cv})$ if 4 probes located as shown:



a.2) Repeat a.1 for 4 probes located as below:



a.3) Repeat a.1 for one probe in center.

(b) Data Preparation

Card Type	Data
(1)	NGC = 0 (circular) A = 1 (radius) B = 0 (dummy) NIC = 0 (raw data)
(2)	Sample Problem 3
(3)	Concentration Raw Data. $\underline{C}$ $\underline{R}$ $\underline{\theta}$ . . . .
(4)	 9999
(5)	Velocity Raw Data. $\underline{V}$ $\underline{R}$ $\underline{\Theta}$ . . . .
(4)	 9999
(6)	It is clear that concentration profile is independent of θ (zero order) and is second order in R: Therefore: MCYOT = -1 MCXOR = 0 The velocity profile appears to be second order in both R and θ, therefore: MVXOR = 0 MVYOT = o
(10)	NST = 1
(11)	PROBLEM 3 - a.1 (4 probes)
(12)	NASEG = 1

Card Type **Data**

(13) Probe Locations

R	θ	Area Seg. No.
.5	0.0	1
.5	1.57080	1
.5	3.14159	1
.5	4.71239	1

(4) 9999

(10) NST = 1

(11) Problem 3 - a.2 {4 probes)

(12) NASEG = 2

(13) Probe Locations

R	θ	Area Seg. No.
.333	0.0	1
.333	3.14159	2
.6666	0.0	1
.6666	3.14159	2

(4) 9999

(10) NST = 1

(11) Problem 3 - a.3 (one probe)

(12) NASEG = 1

(13) Probe Locations

R	θ	Area Seg. No.
0	0	1

(4) 9999

(10) NST = 0 (End of Problem 3)

(1) NGC = 2 (End of Job)

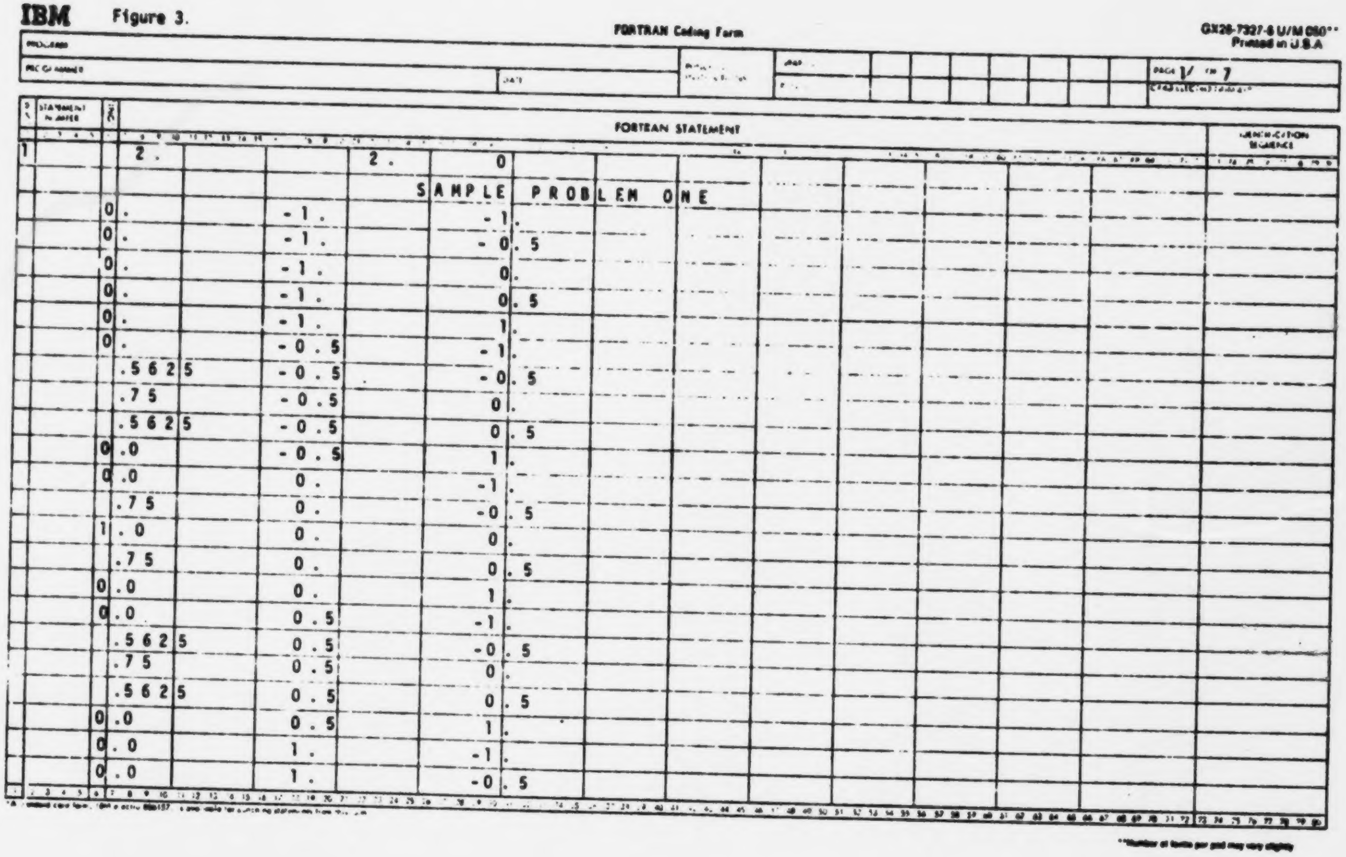
D. SAMPLE CASE INPUT DATA

Completed input data sheets are shown in Figure 3 for the three sample case problems.

E. SAMPLE CASE OUTPUT DATA

Figure 4 is the printed output generated by the program for the three problems specified as the sample cases.

IBM Figure 3.



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PAGE 1/ OF 7

FORTRAN STATEMENT

```
1      2.            2.      0
                        SAMPLE PROBLEM ONE
      0.         -1.        -1.
      0.         -1.        -0.5
      0.         -1.         0.
      0.         -1.         0.5
      0.         -1.         1.
      0.         -0.5       -1.
       .5625     -0.5       -0.5
       .75       -0.5        0.
       .5625     -0.5        0.5
      0.0        -0.5        1.
      0.0         0.        -1.
       .75        0.        -0.5
      1.0         0.         0.
       .75        0.         0.5
      0.0         0.         1.
      0.0         0.5       -1.
       .5625      0.5       -0.5
       .75        0.5        0.
       .5625      0.5        0.5
      0.0         0.5        1.
      0.0         1.        -1.
      0.0         1.        -0.5
```

**Number of forms per pad may vary slightly

IBM

FORTRAN Coding Form

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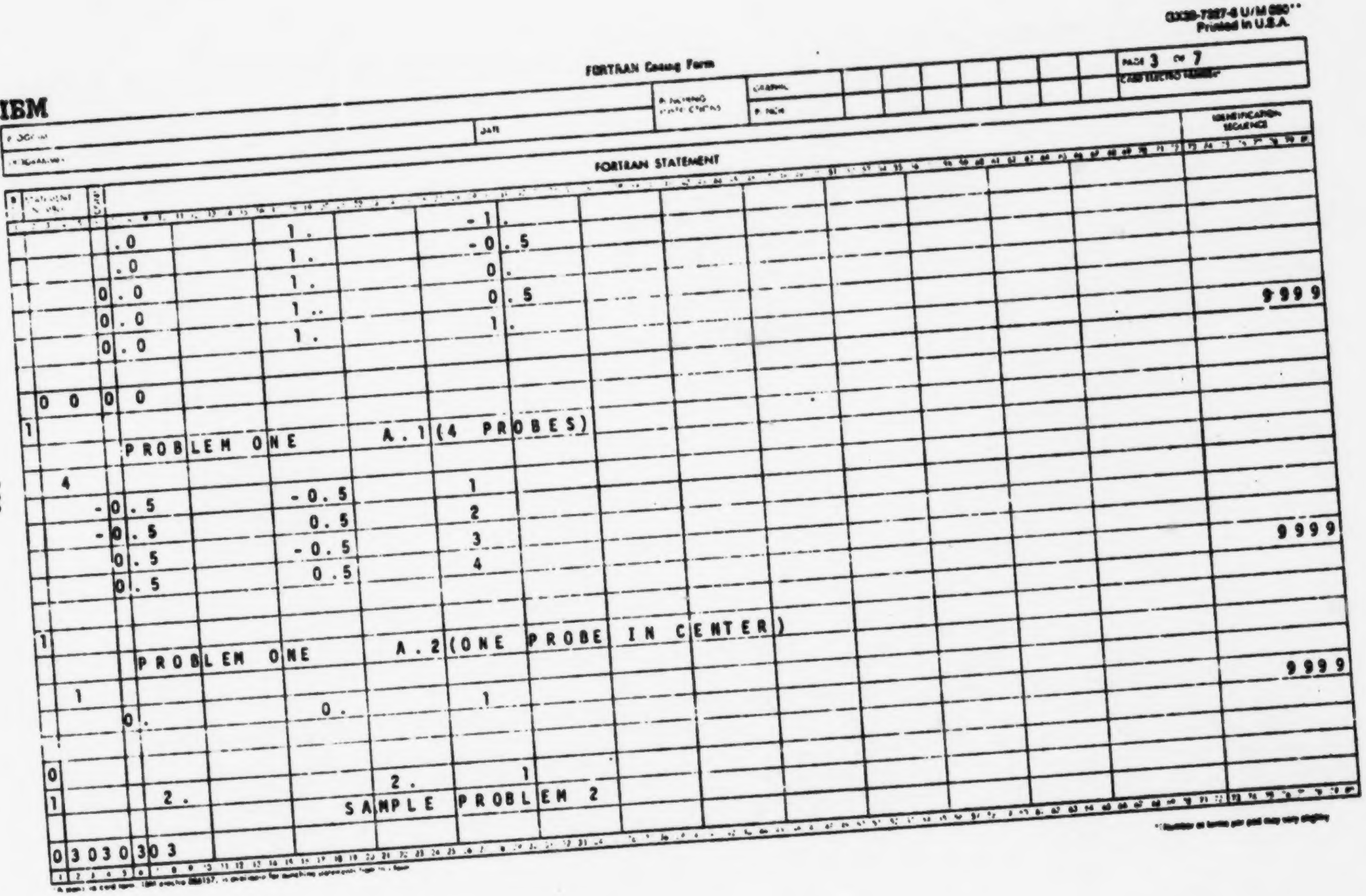
PROGRAM | PUNCHING INSTRUCTIONS | GRAPHIC | PUNCH | PAGE 2 OF 7
PROGRAMMER | DATE | CARD ELECTRO NUMBER*

STATEMENT NUMBER | FORTRAN STATEMENT | IDENTIFICATION SEQUENCE

```
     0.0         1.          0.
     0.0         1.          0.5
     0.0         1.          1.
                                                                            9999
     0.0        -1.         -1.
     0.0        -1.         -0.5
     0.0        -1.          0.
     0.0        -1.          0.5
     0.0        -1.          1.
     0.0        -0.5        -1.
      .5625     -0.5        -0.5
      .75       -0.5         0.
      .5625     -0.5         0.5
     0.0        -0.5         1.
     0.0         0.         -1.
      .75        0.         -0.5
     1.0         0.          0.
      .75        0.          0.5
     0.0         0.          1.
     0.0         0.5        -1.
      .5625      0.5        -0.5
      .75        0.5         0.
      .5625      0.5         0.5
      .0         0.5         1.
```

*A standard card form, IBM electro 888157, is available for punching statements from this form

**Number of forms per pad may vary slightly



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IBM

FORTRAN Coding Form

PAGE 3 OF 7

FORTRAN STATEMENT

IDENTIFICATION SEQUENCE

```
      .0         1.         -1.
      .0         1.         -0.5
     0.0         1.          0.
     0.0         1.          0.5
     0.0         1.          1.                                             9999

 0 0 0 0
1
      PROBLEM ONE     A.1(4 PROBES)
  4
    -0.5        -0.5         1
    -0.5         0.5         2
     0.5        -0.5         3
     0.5         0.5         4                                               9999

1
      PROBLEM ONE     A.2(ONE PROBE IN CENTER)
  1
     0.          0.          1                                               9999

0                     2.      1
1      2.         SAMPLE PROBLEM 2

03030303
```

GX28-7327-6 U/M 050**
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IBM FORTRAN Coding Form

PAGE 4 OF 7

FORTRAN STATEMENT

```
     1.              0.             -1.              0.              0.
     0.             -1.              0.              1.
     1.              0.             -1.              0.              0.
     0.             -1.              0.              1.
1
      PROBLEM 2       A.1(4 PROBES)
  2
     1.          1.        1
     0.          0.5       1
     0.         -0.5       2
    -1.         -1.        2

0                                                                           9999
0      1.                    0
       SAMPLE PROBLEM 3
     2.          0.          0.
     2.          0.          3.14159
     2.0625      0.25        0.
     2.C625       .25        3.14159
     2.25         .5         0.
     2.25         .5         3.14159
     2.5625       .75        0.
     2.5625       .75        3.14159
     3.          1.          0.
```

*A standard card form, IBM electro 888157, is available for punching statements from this form

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FORTRAN Coding Form

GX28-7327-6 U/M 050**
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PAGE 5 OF 7

FORTRAN STATEMENT

```
     3.          1.          3.14159
                                                                            9999
 22500.          0.          0.
 22500.          0.          1.57080
 22500.          0.          3.14159
 22500.          0.          4.71239
 22500.          0.          6.28318
 16875.5          .25        0.
 16522.34375      .25        1.57080
 16400.625        .25        3.14159
 16522.34375      .25        4.71239
 16875.5          .25        6.28318
 11250.           .5         0.
 10781.125        .5         1.57080
 10622.5          .5         3.14159
 10781.125        .5         4.71239
 11250.           .5         6.28318
  5622.5          .75        0.
  5273.34375      .75        1.57080
  5155.625        .75        3.14159
  5273.34375      .75        4.71239
  5622.5          .75        6.28318
     0.          1.          0.
     0.          1.          1.57080
```

*A standard card form, IBM electro 888157, is available for punching statements from this form

**Number of forms per pad may vary slightly

IBM FORTRAN Coding Form

GX28-7327-6 U/M 050**
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PROGRAM
PROGRAMMER
DATE
PUNCHING INSTRUCTIONS
GRAPHIC
PUNCH
PAGE 6 OF 7
CARD ELECTRO NUMBER*

STATEMENT NUMBER | FORTRAN STATEMENT | IDENTIFICATION SEQUENCE

```
     0.          1.          3.14159
     0.          1.          4.71239
     0.          1.          6.28318
                                                                            9999
 0-1 0 0
1
      PROBLEM 3      A.1(4 PROBES)
001
      .5         0.0       1
      .5         1.57080   1
      .5         3.14159   1
      .5         4.71239
                                                                            9999
1
      PROBLEM 3      A.2 (4 PROBES)
002
      .3333      0.        1
      .3333      3.14159   2
      .6666      0.        1
      .6666      3.14159   2
                                                                            9999
1
      PROBLEM 3      A.3 (ONE PROBE)
001
```

*A standard card form, IBM electro 888157, is available for punching statements from this form

**Number of forms per pad may vary slightly

IBM

FORTRAN Coding Form

GX28-7327-6 U/M 050**
Printed in U.S.A.

PROGRAM		PAGE 7 OF 7
PROGRAMMER	DATE	CARD ELECTRO NUMBER*

C	STATEMENT NUMBER	CONT.	FORTRAN STATEMENT	IDENTIFICATION SEQUENCE

```
     0.0         0.0      1
                                                                            9999
0
2
```

*A standard card form, IBM electro 888157, is available for punching statements from this form

**Number of forms per pad may vary slightly

```
NEW DATA.
NO. OF CON. DATA CARDS  15      NO. OF VEL. DATA CARDS  25
INPUT AND OUTPUT ARRY OF CROUT ROUTIN FOR CONCENTRATION

0.250000E 02 0.0          0.125000E 02 0.0          0.0          0.0          0.125000E 02 0.0          0.625000E 01 0.625000E 01
0.0          0.125000E 02 0.0          0.0          0.0          0.0          0.0          0.625000E 01 0.0          0.0
0.125000E 02 0.0          0.106250E 02 0.0          0.0          0.0          0.625000E 01 0.0          0.531250E 01 0.937500E 00
0.0          0.0          0.0          0.125000E 02 0.0          0.625000E 01 0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.625000E 01 0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.625000E 01 0.0          0.531250E 01 0.0          0.0          0.0          0.0
0.125000E 02 0.0          0.625000E 01 0.0          0.0          0.0          0.106250E 02 0.0          0.531250E 01 0.937500E 00
0.0          0.625000E 01 0.0          0.0          0.0          0.0          0.0          0.531250E 01 0.0          0.0
0.625000E 01 0.0          0.531250E 01 0.0          0.0          0.0          0.531250E 01 0.0          0.451563E 01 0.140625E 00

0.250000E 02 0.0          0.500000E 00 0.0          0.0          0.0          0.500000E 00 0.0          0.250000E 00 0.100000E 01
0.0          0.125000E 02 0.0          0.0          0.0          0.0          0.0          0.500000E 00 0.0          0.0
0.125000E 02 0.0          0.437500E 01 0.0          0.0          0.0          0.0          0.0          0.500000E 00-0.100000E 01
0.0          0.0          0.0          0.125000E 02 0.0          0.500000E 00 0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.625000E 01 0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.625000E 01 0.0          0.218750E 01 0.0          0.0          0.0          0.0
0.125000E 02 0.0          0.0          0.0          0.0          0.0          0.437500E 01 0.0          0.500000E 00-0.100000E 01
0.0          0.625000E 01 0.0          0.0          0.0          0.0          0.0          0.218750E 01 0.0          0.0
0.625000E 01 0.0          0.218750E 01 0.0          0.0          0.0          0.218750E 01 0.0          0.765625E 00 0.100000E 01

INPUT AND OUTPUT ARRY OF CROUT ROUTIN FOR VEL.

0.250000E 02 0.0          0.125000E 02 0.0          0.0          0.0          0.125000E 02 0.0          0.625000E 01 0.625000E 01
0.0          0.125000E 02 0.0          0.0          0.0          0.0          0.0          0.625000E 01 0.0          0.0
0.125000E 02 0.0          0.106250E 02 0.0          0.0          0.0          0.625000E 01 0.0          0.531250E 01 0.937500E 00
0.0          0.0          0.0          0.125000E 02 0.0          0.625000E 01 0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.625000E 01 0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.625000E 01 0.0          0.531250E 01 0.0          0.0          0.0          0.0
0.125000E 02 0.0          0.625000E 01 0.0          0.0          0.0          0.106250E 02 0.0          0.531250E 01 0.937500E 00
0.0          0.625000E 01 0.0          0.0          0.0          0.0          0.0          0.531250E 01 0.0          0.0
0.625000E 01 0.0          0.531250E 01 0.0          0.0          0.0          0.531250E 01 0.0          0.451563E 01 0.140625E 00

0.250000E 02 0.0          0.500000E 00 0.0          0.0          0.0          0.500000E 00 0.0          0.250000E 00 0.100000E 01
0.0          0.125000E 02 0.0          0.0          0.0          0.0          0.0          0.500000E 00 0.0          0.0
0.125000E 02 0.0          0.437500E 01 0.0          0.0          0.0          0.0          0.0          0.500000E 00-0.100000E 01
0.0          0.0          0.0          0.125000E 02 0.0          0.500000E 00 0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.0          0.625000E 01 0.0          0.0          0.0          0.0          0.0
0.0          0.0          0.0          0.625000E 01 0.0          0.218750E 01 0.0          0.0          0.0          0.0
0.125000E 02 0.0          0.0          0.0          0.0          0.0          0.437500E 01 0.0          0.500000E 00-0.100000E 01
0.0          0.625000E 01 0.0          0.0          0.0          0.0          0.0          0.218750E 01 0.0          0.0
0.625000E 01 0.0          0.218750E 01 0.0          0.0          0.0          0.218750E 01 0.0          0.765625E 00 0.100000E 01
```

Figure 4. Sample Case Output

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```
                              SAMPLE PROBLEM ONE

          STACK GEOMETRY
               RECTANGULAR-A(FT.)=     2.00000  B(FT.)=    2.00000
                      PROFILE DATA
        CONCENTRATION COEFFICIENTS
   C 1, 1= 0.1000000000E 01
   C 2, 1= 0.0
   C 3, 1=-0.1000000000E 01
   C 1, 2= 0.0
   C 2, 2= 0.0
   C 3, 2= 0.0
   C 1, 3=-0.1000000000E 01
   C 2, 3= 0.0
   C 3, 3= 0.1000000000E 01

        VELOCITY,COEFFICIENTS
   V 1, 1= 0.1000000000E 01
   V 2, 1= 0.0
   V 3, 1=-0.1000000000E 01
   V 1, 2= 0.0
   V 2, 2= 0.0
   V 3, 2= 0.0
   V 1, 3=-0.1000000000E 01
   V 2, 3= 0.0
   V 3, 3= 0.1000000000F 01
```

```
                          RESULTS OF PROFILE CORRELATION
                                      INPUT                           PREDICTED
   2X/A          2Y/B
                          CONCENTRATION    VELOCITY    CONCENTRATION    VELOCITY
-1.00000      -1.00000       0.0                          0.0           0.0
-1.00000      -0.50000       0.0                          0.0           0.0
-1.00000       0.0           0.0                          0.0           0.0
-1.00000       0.50000       0.0                          0.0           0.0
-1.00000       1.00000       0.0                          0.0           0.0
-0.50000      -1.00000       0.0                          0.0           0.0
-0.50000      -0.50000       0.56250                      0.56250       0.56250
-0.50000       0.0           0.75000                      0.75000       0.75000
-0.50000       0.50000       0.56250                      0.56250       0.56250
-0.50000       1.00000       0.0                          0.0           0.0
 0.0          -1.00000       0.0                          0.0           0.0
 0.0          -0.50000       0.75000                      0.75000       0.75000
 0.0           0.0           1.00000                      1.00000       1.00000
 0.0           0.50000       0.75000                      0.75000       0.75000
 0.0           1.00000       0.0                          0.0           0.0
 0.50000      -1.00000       0.0                          0.0           0.0
 0.50000      -0.50000       0.56250                      0.56250       0.56250
 0.50000       0.0           0.75000                      0.75000       0.75000
 0.50000       0.50000       0.56250                      0.56250       0.56250
 0.50000       1.00000       0.0                          0.0           0.0
 1.00000      -1.00000       0.0                          0.0           0.0
 1.00000      -0.50000       0.0                          0.0           0.0
 1.00000       0.0           0.0                          0.0           0.0
 1.00000       0.50000       0.0                          0.0           0.0
 1.00000       1.00000       0.0                          0.0           0.0
```

```
                                RESULTS OF PROFILE CORRELATION
                                          INPUT                          PREDICTED
   2X/A           2Y/L
                              CONCENTRATION    VELOCITY    CONCENTRATION    VELOCITY
  -1.00000      -1.00000                         0.0          0.0             0.0
  -1.00000      -0.50000                         0.0          0.0             0.0
  -1.00000       0.0                             0.0          0.0             0.0
  -1.00000       0.50000                         0.0          0.0             0.0
  -1.00000       1.00000                         0.0          0.0             0.0
  -0.50000      -1.00000                         0.0          0.0             0.0
  -0.50000      -0.50000                         0.56250      0.56250         0.56250
  -0.50000       0.0                             0.75000      0.75000         0.75000
  -0.50000       0.50000                         0.56250      0.56250         0.56250
  -0.50000       1.00000                         0.0          0.0             0.0
   0.0          -1.00000                         0.0          0.0             0.0
   0.0          -0.50000                         0.75000      0.75000         0.75000
   0.0           0.0                             1.00000      1.00000         1.00000
   0.0           0.50000                         0.75000      0.75000         0.75000
   0.0           1.00000                         0.0          0.0             0.0
   0.50000      -1.00000                         0.0          0.0             0.0
   0.50000      -0.50000                         0.56250      0.56250         0.56250
   0.50000       0.0                             0.75000      0.75000         0.75000
   0.50000       0.50000                         0.56250      0.56250         0.56250
   0.50000       1.00000                         0.0          0.0             0.0
   1.00000      -1.00000                         0.0          0.0             0.0
   1.00000      -0.50000                         0.0          0.0             0.0
   1.00000       0.0                             0.0          0.0             0.0
   1.00000       0.50000                         0.0          0.0             0.0
   1.00000       1.00000                         0.0          0.0             0.0
 STANDARD DEVIATION IN C= 0.0               IN V= 0.0
```

```
               PROBLEM ONE     A.1(4 PROBES)

             PROBE LOCATION DATA
     NUMBER OF PROBES=  4
  2X/A           2Y/B
                           AREA SEGMENT NUMBER
  -0.50000      -0.50000          1
  -0.50000       0.50000          2
   0.50000      -0.50000          3
   0.50000       0.50000          4

        ACTUAL                                          COMPUTED
AVERAGE V        AVERAGE C*V          AVERAGE V      AVERAGE C*V      AVERAGE C
  0.444              0.264                 0.563           0.316           0.563
 ERROR IN CVBAR=          0.11236
 ERROR IN CBAR*VBAR=      0.11236
 ERROR IN VBAR=          0.26562


                PROBLEM ONE     A.2(ONE PROBE IN CENTER)

             PROBE LOCATION DATA
     NUMBER OF PROBES=  1
  2X/A           2Y/B
                           AREA SEGMENT NUMBER
  0.0            0.0              1


        ACTUAL                                          COMPUTED
AVERAGE V        AVERAGE C*V          AVERAGE V      AVERAGE C*V      AVERAGE C
  0.444              0.264                 1.000           1.000           1.000
 ERROR IN CVBAR=          2.51562
 ERROR IN CBAR*VBAR=      2.51562
 ERROR IN VBAR=          1.25000
```

```
                              SAMPLE PROBLEM 2

            STACK GEOMETRY
                  RECTANGULAR-A(FT.)=     2.00000  B(FT.)=     2.00000
                         PROFILE DATA
        CONCENTRATION COEFFICIENTS
    C 1, 1= 0.100000000E 01
    C 2, 1= 0.0
    C 3, 1=-0.100000000E 01
    C 1, 2= 0.0
    C 2, 2= 0.0
    C 3, 2= 0.0
    C 1, 3=-0.100000000E 01
    C 2, 3= 0.0
    C 3, 3= 0.100000000E 01

        VELOCITY,COEFFICIENTS
    V 1, 1= 0.100000000E 01
    V 2, 1= 0.0
    V 3, 1=-0.100000000E 01
    V 1, 2= 0.0
    V 2, 2= 0.0
    V 3, 2= 0.0
    V 1, 3=-0.100000000E 01
    V 2, 3= 0.0
    V 3, 3= 0.100000000E 01
```

```
                PROBLEM 2      A.1(4 PROBES)

                PROBE LOCATION DATA
   NUMBER OF PROBES=  4
  2X/A          2Y/B
                            AREA SEGMENT NUMBER
  1.00000        1.00000          1
  0.0            0.50000          1
  0.0           -0.50000          2
 -1.00000       -1.00000          2


        ACTUAL                                                COMPUTED
AVERAGE V       AVERAGE C*V               AVERAGE V        AVERAGE C*V       AVERAGE C
   0.444           0.284                     0.375             0.281              0.375
 ERROR IN CVBAR=           0.01123
 ERROR IN CBAR*VBAR=       0.50562
 ERROR IN VBAR=            0.15625
```

```
NEW DATA.
NO OF CON. DATA CARDS  10      NO. OF VEL. DATA CARDS  25
INPUT AND OUTPUT ARRY OF CROUT ROUTIN FOR CONCENTRATION

 0.100000E 02 0.500000E 01 0.375000E 01 0.227500E 02 0.500000E 01 0.375000E 01 0.312500E 01 0.131250E 02 0.375000E 01 0.312500E 01
 0.27[illegible] 01 0.102656E 02

 0.1[illegible] 02 0.500000E 00 0.375000E 00 0.200000E 01 0.500000E 01 0.125000E 01 0.100000E 01 0.0          0.375000E 01 0.125000E 01
 0.109375E 00 0.100000E 01

INPUT AND OUTPUT ARRY OF CROUT ROUTIN FOR VEL.

 [illegible] 02 [illegible] 02 0.934[illegible] 01 0.125000E 02 0.624999E 01 0.468749E 01 0.937500E 01 0.468749E 01 0.351562E 01 0.277344E 05
 [illegible] 02 0.434603[illegible] 01 0.780306[illegible] 01 0.624999E 01 0.468749E 01 0.390623E 01 0.468749E 01 0.351562E 01 0.292968E 01 0.127190E 05
 [illegible] 0[illegible] 0.7[illegible] 01 0.691127E 01 0.468749E 01 0.390623E 01 0.345701E 01 0.351562E 01 0.29296[illegible] 01 0.254276E 01 0.103702E 05
 0.1[illegible] 02 0.6[illegible] 01 0.468749E 01 0.937500E 01 0.468749E 01 0.351562E 01 0.761250E 01 0.390624E 01 0.292968E 01 0.683594E 04
 0.[illegible] 01 0.4[illegible] 01 0.390[illegible] 01 0.468749E 01 0.351562E 01 0.292968E 01 0.390624E 01 0.292968E 01 0.244140E 01 0.341796E 04
 0.46[illegible] 01 0.390[illegible] 01 0.345761E 01 0.351562E 01 0.292968E 01 0.259277E 01 0.292968E 01 0.244140E 01 0.216063E 01 0.256056E 04
 0.9[illegible] 0[illegible] 0.4[illegible] 0[illegible] 0.351[illegible] 01 0.7[illegible] 01 0.390624E 01 0.292[illegible] 01 0.691406E 01 0.345703E 01 0.254277E 01 0.340332E 04
 [illegible] 0[illegible] 0.3[illegible] 0[illegible] 0.2[illegible] 01 0.[illegible]0624E 01 0.292968E 01 0.244140E 01 0.345703E 01 0.259277E 01 0.216064E 01 0.170166E 04
 [illegible]-01 0.29[illegible] 01 0.[illegible] 01 0.292968E 01 0.244140E 01 0.216063E 01 0.259277E 01 0.216064E 01 0.191216E 01 0.128607E 04

 0.[illegible] 02 0.4979[illegible] 00 0.37[illegible] 00 0.500000E 00 0.250000E 00 0.187499E 00 0.375000E 00 0.187500E 00 0.140625E 00 0.224999E 04
 [illegible] [illegible] [illegible] 00 0.164117E-01 0.49[illegible] 00 0.498829E 00 0.750695E-02 0.374507E 00 0.373529E 00 0.167500E 00
 [illegible] 01 [illegible] 01 [illegible] 00-[illegible]-0[illegible]-0.2[illegible]142E-01 0.4[illegible] 00-0.515116E-01-0.153674E-01 0.370692E 00-0.220447E 00
 [illegible] 02 0.[illegible]-01-0.1[illegible]-01 0.3[illegible] 01 0.49[illegible]73E 00 0.372[illegible] 00 0.100013E 01 0.456219E 00 0.373458E 00-0.224490E 04
 [illegible] 01 0.15[illegible] 01-0.55[illegible]-0[illegible] 0.15[illegible] 01 0.349653E 00 0.445369E 00 0.100247E-01 0.999352E 00 0.994650E 00-0.100112E 04
 [illegible] 01 0.157[illegible] 01 0.135[illegible] 00 0.116404E 01 0.347811E 00 0.332074E-01-0.700457E-01-0.215023E-01 0.993905E 00 0.100126E 04
 0.937500E 01 0.[illegible]935[illegible]-01-0.134147E-01 0.312379E 01 0.460543E-02-0.233138E-02 0.273136E 00 0.496527E 00 0.373725E 00-0.949707E-01
 0.468749E 01 0.118422E 01-0.415916E-02 0.154[illegible] 01 0.369395E 00-0.715194E-03 0.135619E 00 0.347235E-01 0.956704E 00 0.100101E 04
 0.351562E 01 0.111[illegible] 01 0.1[illegible]173[illegible] 00 0.11[illegible]645E 01 0.347515E 00 0.330944E-01 0.102079E 00 0.2[illegible]-01 0.293259E-02-0.100112E 04
```

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```
SAMPLE PROBLEM 3


        STACK GEOMETRY
              CIRCULAR-RADIUS=    1.00000FT.
                     PROFILE DATA
     CONCENTRATION COEFFICIENTS
 C 1, 1= 0.2000000000E 01
 C 1, 2= 0.0
 C 1, 3= 0.1000000000E 01


     VELOCITY,COEFFICIENTS
 V 1, 1= 0.2249957066E 04
 V 2, 1= 0.1875000000E 00
 V 3, 1=-0.2209472668E 00
 V 1, 2=-0.2249900158E 04
 V 2, 2=-0.1001124272E 04
 V 3, 2= 0.1001268123E 04
 V 1, 3=-0.9457070316E-01
 V 2, 3= 0.1001008846E 04
 V 3, 3=-0.1001119838E 04


                        RESULTS OF PROFILE CORRELATION
                                   INPUT                      PREDICTED
   S/A         THETA/2PI
                         CONCENTRATION  VELOCITY   CONCENTRATION  VELOCITY
  0.0           0.0          2.00000
  0.0           0.50000      2.00000                  2.00000     2249.95706
  0.25000       0.0          2.06250                  2.00000     2250.07515
  0.25000       0.50000      2.06250                  2.06250     1667.50610
  0.50000       0.0          2.25000                  2.06250     1640.62158
  0.50000       0.50000      2.25000                  2.25000     1125.01318
  0.75000       0.0          2.56250                  2.25000     1062.48413
  0.75000       0.50000      2.56250                  2.56250      562.50854
  1.00000       0.0          3.00000                  2.56250      515.61279
  1.00000       0.50000      3.00000                  3.00000       -0.00806
                                                      3.00000        0.00774
```

```
                              RESULTS OF PROFILE CORRELATION
                                         INPUT                          PREDICTED
    R/A          THETA/2PI
                              CONCENTRATION     VELOCITY     CONCENTRATION     VELOCITY
    0.0             0.0                        2250.00000       2.00000     2249.98706
    0.0             0.16415                    2250.00000       2.00000     2250.01392
    0.0             0.50000                    2250.00000       2.00000     2250.02515
    0.0             0.75000                    2250.00000       2.00000     2250.00317
    0.0             1.00000                    2250.00000       2.00000     2249.95337
    0.25000         0.0                        1687.50000       2.06250     1687.50610
    0.25000         0.25000                    1652.34375       2.06250     1652.34351
    0.25000         0.50000                    1640.62500       2.06250     1640.62158
    0.25000         0.75000                    1652.34375       2.06250     1652.34058
    0.25000         1.00000                    1687.50000       2.06250     1687.49951
    0.50000         0.0                        1125.00000       2.25000     1125.01316
    0.50000         0.25000                    1078.12500       2.25000     1078.11546
    0.50000         0.50000                    1062.50000       2.25000     1062.48413
    0.50000         0.75000                    1076.12500       2.25000     1076.11914
    0.50000         1.00000                    1125.00000       2.25000     1125.02002
    0.75000         0.0                         562.50000       2.56250      562.50854
    0.75000         0.25000                     527.34375       2.56250      527.33594
    0.75000         0.50000                     515.62500       2.56250      515.61279
    0.75000         0.75000                     527.34375       2.56250      527.33936
    0.75000         1.00000                     562.50000       2.56250      562.51469
    1.00000         0.0                           0.0           3.00000       -0.00806
    1.00000         0.25000                       0.0           3.00000        0.00491
    1.00000         0.50000                       0.0           3.00000        0.00774
    1.00000         0.75000                       0.0           3.00000        0.00098
    1.00000         1.00000                       0.0           3.00000       -0.01587
 STANDARD DEVIATION IN C= 0.0                  IN V= 0.14303E-01
```

```
PROBLEM 3     A.1(4 PROBES)


          PROBE LOCATION DATA
   NUMBER OF PROBES=  4
R/A         THETA/2PI
                        AREA SEGMENT NUMBER
0.50000       0.0              1
0.50000       0.25000          1
0.50000       0.50000          1
0.50000       0.75000          1




          ACTUAL                                           COMPUTED
  AVERAGE V       AVERAGE C*V           AVERAGE V      AVERAGE C*V        AVERAGE C
   722.210         1658.326              1085.933        2443.349            2.250
   ERROR IN CVBAR=          0.47338
   ERROR IN CBAR*VBAR=      0.47338
   ERROR IN VBAR=          0.50361
```

```
                    PROBLEM 3    A.2 (4 PROBES)

              PROBE LOCATION DATA
      NUMBER OF PROBES= 4
   R/A          THETA/2PI
                           AREA SEGMENT NUMBER
   0.33330      0.0              1
   0.33330      0.50000          2
   0.66660      0.0              1
   0.66660      0.50000          2


      ACTUAL                                          COMPUTED
AVERAGE V       AVERAGE C*V               AVERAGE V   AVERAGE C*V     AVERAGE C
 722.218          1658.326                 1007.333     2436.938        2.278
 ERROR IN CVBAR=          0.46952
 ERROR IN CBAR*VBAR=      0.50719
 ERROR, IN VBAR=          0.51939

                    PROBLEM 3    A.3 (ONE PROBE)

              PROBE LOCATION DATA
      NUMBER OF PROBES= 1
   R/A          THETA/2PI
                           AREA SEGMENT NUMBER
   0.0          0.0              1


      ACTUAL                                          COMPUTED
AVERAGE V       AVERAGE C*V               AVERAGE V   AVERAGE C*V     AVERAGE C
 722.216          1658.326                 2249.987     4499.973        2.000
 ERROR IN CVBAR=          1.71356
 ERROR IN CBAR*VBAR=      1.71356
 ERROR IN VBAR=           2.11538
```

F. RESULTS AND CONCLUSIONS

a) Problem 1

The concentration polynomial was determined to be

$$C = 1 + (0)\left(\frac{Y}{B/2}\right) - (1)\left(\frac{Y}{B/2}\right)^2 + \left[0 + (0)\left(\frac{Y}{B/2}\right) + (0)\left(\frac{Y}{B/2}\right)^2\right]\left(\frac{X}{A/2}\right)$$

$$+ \left[-1 + (0)\left(\frac{Y}{B/2}\right) + (1)\left(\frac{Y}{B/2}\right)^2\right]\left(\frac{X}{A/2}\right)^2$$

where $A = B = 2$

$C = 1 - Y^2 - (+1 - Y^2)\,X^2$ or $C = (1-X^2)\,(1-Y^2)$

Similarly, the velocity polynomial was found to be:

$V = (1-X^2)\,(1-Y^2)$

Standard deviation in C = 0 (perfect fitting)
Standard deviation in V = 0 (perfect fitting)
Actual average velocity = .444
Actual average C*V (emission rate) = .284

(1) For the four probes case:
Predicted average velocity = .563 --- error of 26.6%
Predicted emission rate = .316 --- error of 11.2%
Predicted average concentration = .563

(2) For the case of one probe in center
Predicted average velocity = 1, error = 125%
Predicted emission rate = 1, error = 251%

b) Problem 2

Actual average velocity = .444
Actual emission rate = .284

(1) For the 4 probes case:

Predicted average velocity = .375 , error of 15.6%
Predicted emission rate = .281 , error 1.1%

Note that if the emission rate is to be found by multiplying the average concentration and average velocity, the predicted error is 50.5%

c) Problem 3

Concentration polynomial $C = 2 + (1)\left(\frac{R}{A}\right)^2$

$A = 1$

therefore: $C = 2 + R^2$

Velocity polynomial

$$V = 2249.98 + (.1875)\left(\frac{\Theta}{2\pi}\right) - .2209\left(\frac{\theta}{2\pi}\right)^2$$

$$+ \left[- 2249.90 - 1001\left(\frac{\theta}{2\pi}\right) + 1001\left(\frac{\theta}{2\pi}\right)^2\right]\left(\frac{R}{A}\right)$$

$$+ \left[-0.0949 + 1001\left(\frac{\theta}{2\pi}\right) - 1001\left(\frac{\theta}{2\pi}\right)^2\right]\left(\frac{R}{A}\right)^2$$

where $A = 1$

Considering the order of magnitudes of the coefficients, the polynomial can be approximated as:

$$V = 2250 - \left[2250 + 1000\left(\frac{\theta}{2\pi}\right) - 1000\left(\frac{\theta}{2\pi}\right)^2\right] R + \left[1000\left(\frac{\theta}{2\pi}\right) - 1000\left(\frac{\theta^2}{2\pi}\right)\right] R^2 \quad \underline{\text{or}}$$

$$V = 1000\left[1 - R\right]\left\{2.25 + R\left[\left(\frac{\theta}{2\pi}\right)^2 - \left(\frac{\theta}{2\pi}\right)\right]\right\}$$

where at $R = 1$ $V = 0$ and at $\theta = 0$ and $\theta = 2\pi$
$V = 2250\,[1-R]$

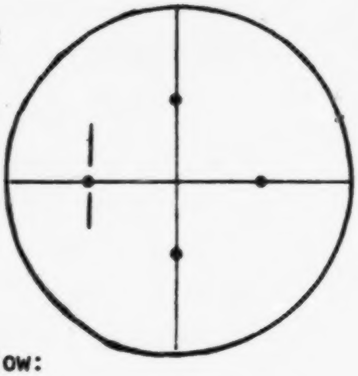
Standard deviation in C = 0 (perfect fitting)

Standard deviation in V = 0.014

Actual average velocity = 722.218

Actual emission rate = 1658.326

(1) For the case of 4 probes shown below:



Predicted average velocity = 1085.933

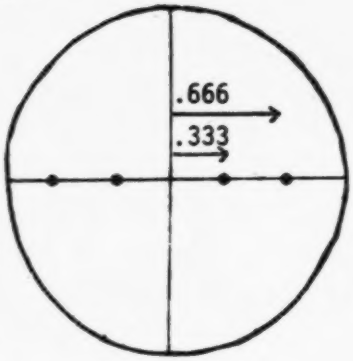
error = 50%

Predicted emission rate = 2443.349

error = 47%

Predicted average concentration = 2.25

(2) For the case of 4 probes as shown below:



Predicted average velocity = 1097.333

error = 51.9%

Predicted emission rate = 2436.938

error = 46.9%

Note that if the emission is based on the product of average velocity and on average concentration error is equal to 50.7%

(3) For the case of one probe in center:

Predicted average velocity = 224.987

error = 211%

Predicted emission rate = 4499.973

error = 171%

VI. MISCELLANEOUS OPERATIONAL INFORMATION

A. PROGRAM CAPACITY

The program requires approximately 200K core locations during loading and execution on the IBM 360/65 computer. Actual program length including buffers is approximately 114K.

The execution time for the program depends primarily on the number of maxima and minima in planes of constant Y (or Θ) and X (or R) in the concentration and velocity data. A typical operating case such as the sample problems in Section VI requires approximately 11.63 minutes of execution time (CPU time is 19.50 seconds for compilation and 4.31 seconds for execution) on the IBM 360/65.

No tapes are required by the program.

B. PROGRAM MODIFICATIONS

1. The maximum number of input concentration or velocity data or number of probes can be increased or decreased by modifying the dimension statements in the main program nad COEFG subroutine. (Variables CON, VEL, XORR, YORT, XRV, YTV in Main and variables Z, X, Y in COEFG subroutine.)

2. The maximum order in X (or R) or Y (or θ) of the velocity and concentration polynomials can be changed by modifying the DIMENSION statements in the main program and all subroutines. (Variables; C(M,N), V(M,N), AR(M*N, M*N+1), and CORV(M,N) where M and N are the max. order in Y (or θ) and X (or R) respectively.)

3. The present program computes the emission, velocity, and concentration on the basis of EQUAL AREA STRATEGY (NST = 1, PRBANS Routine). The values 2, 3, 4, and 5 of NST are reserved for future expansion for using different strategies (it is evident that for this purpose, additional routines must be added). Note that the user is able to use the present program for any kind of strategy, provided he prepares the input data based on the equal area strategy.

4. The maximum number of probes and equal area segments can be changed by modifying the dimension statements in the main program and PRBANS subroutine. (Variables; XORR, YORT, IARSEG, and IUSEG).

C. JCL REQUIREMENTS

The following JCL Statements applied when the program was compiled and executed for the test sample case. Since JCL is strongly dependent on both machine and facilities, all necessary changes should be exercised where appropriate by consultation with the systems people at any new facility.

The following JCL Statements are used:

```
//        Job Card
// EXEC FØRTGCG,PARM.FØRT=NØLIST,TIME.GØ=20
//FØRT.SYSIN DD *
  {MAIN PROGRAM
  {SUBROUTINES
/*
//SYSDUMP DD DUMMY
//GO.SYSIN DD *
  {INPUT
  {DATA
/*
```

EXHIBITS

EXHIBIT A	Over-All Program Structure
EXHIBIT B	Description of Principal Parameters
EXHIBIT C	Main Routine
EXHIBIT D	Subroutine COEFG
EXHIBIT E	Subroutine CROUT
EXHIBIT F	Subroutine CVINT
EXHIBIT G	Subroutine PRBANS
EXHIBIT H	Function POLGEN
EXHIBIT I	The CROUT Method for Solving Sets of Linear Simultaneous Equations
EXHIBIT J	Source Listings

EXHIBIT A

OVERALL PROGRAM STRUCTURE

The program consists of a main routine, four subroutines (COEFG, CROUT, CVINT, and PRBANS), and a function subprogram POLGEN.

The program is written in FORTRAN IV. The modular tree diagram of the program is shown in Figure 5.

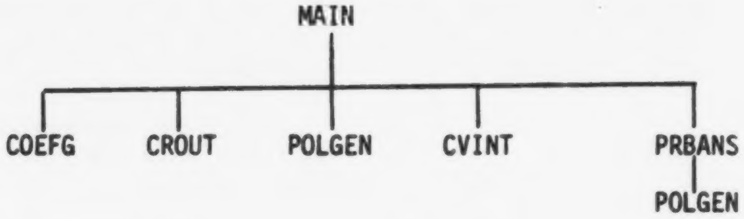

MAIN
COEFG
CROUT
POLGEN
CVINT
PRBANS
POLGEN

Figure 5. Modular Tree Diagram

EXHIBIT B

DESCRIPTION OF PRINCIPAL PARAMETERS

A) INPUT CODES:

Code	Value	Meaning
1. NGC	0	Circular Cross sectioned duct
	1	Rectangular Cross sectioned duct
	2	End of Job.
2. NIC	0	Raw Data Points Submitted for actual duct profiles.
	1	Polynomial coefficients submitted for duct profiles
3. MCXOR	≤ 4	Number of maximum and minimum points in concentration profile in x if rectangular duct or R if circular duct. (see Note 5 - input data form)
4. MCYOT	≤ 4	Number of maximum and minimum points in concentration profile in y if rectangular duct or Θ if circular duct. (see Note 5 - input data form)
5. MVXOR	≤ 4	Number of maximum and minimum points in velocity profile in x if rectangular duct or R if circular duct. (see Note 5 - input data form)
6. MVYOT	≤ 4	Number of maximum and minimum points in velocity profile in y if rectangular duct or Θ if circular duct. (see Note 5 - input data form)

NOTE: Items 3 - 6 are specified only if NIC = 0

Code	Value	Meaning
7. NST	0	Another duct (new duct)
	1	Equal area strategy code
	2	CALL EXIT (FUTURE EXPANSION)
	3	CALL EXIT (FUTURE EXPANSION)
	4	CALL EXIT (FUTURE EXPANSION)
	5	CALL EXIT (FUTURE EXPANSION)
8. NASEQ	≤ 300	Number of equal area segments.

B) OTHER INPUT DATA

If NGC = 0   A - Radius of circle
= 1   A, B - Rectangle Dimensions in x and y directions respectively

If NCI = 0   (CON(I), XORR(I), YORT(I))
(VEL(I), XRV(I), YTV(I))
Concentration and velocity at x, y or R, θ (Theta)

If NCI = 1   ((C(J,I), J=1,MC), I=1,NC)
((V(J,I), J=1,MV), I=1,NV)

where:
C(J,I), and V(J,I) are the coefficients of $\left(\frac{x}{A/2}\right)^{I-1} \left(\frac{y}{B/2}\right)^{J-1}$
or $\left(\frac{R}{A}\right)^{I-1} \left(\frac{\theta}{2\pi}\right)^{J-1}$ in the concentration and velocity polynomials respectively.

MC, MV is the order +1 of polynomial in y or θ

NC, NV is the order +1 of polynomial in x or R

(XORR(I), YORT(I), IARSEG(I))   Probe location coordinates and area segment number.

C) OTHER PARAMETERS (NOT INPUT)

NCP: No. of concentration data points ($\leq$300)

NVP: No. of velocity data points ($\leq$300)

NPR: No. of probes ($\leq$300)

NARM: Error indicator for the duct profile
  NARM = 0   no error
  NARM = 1   error

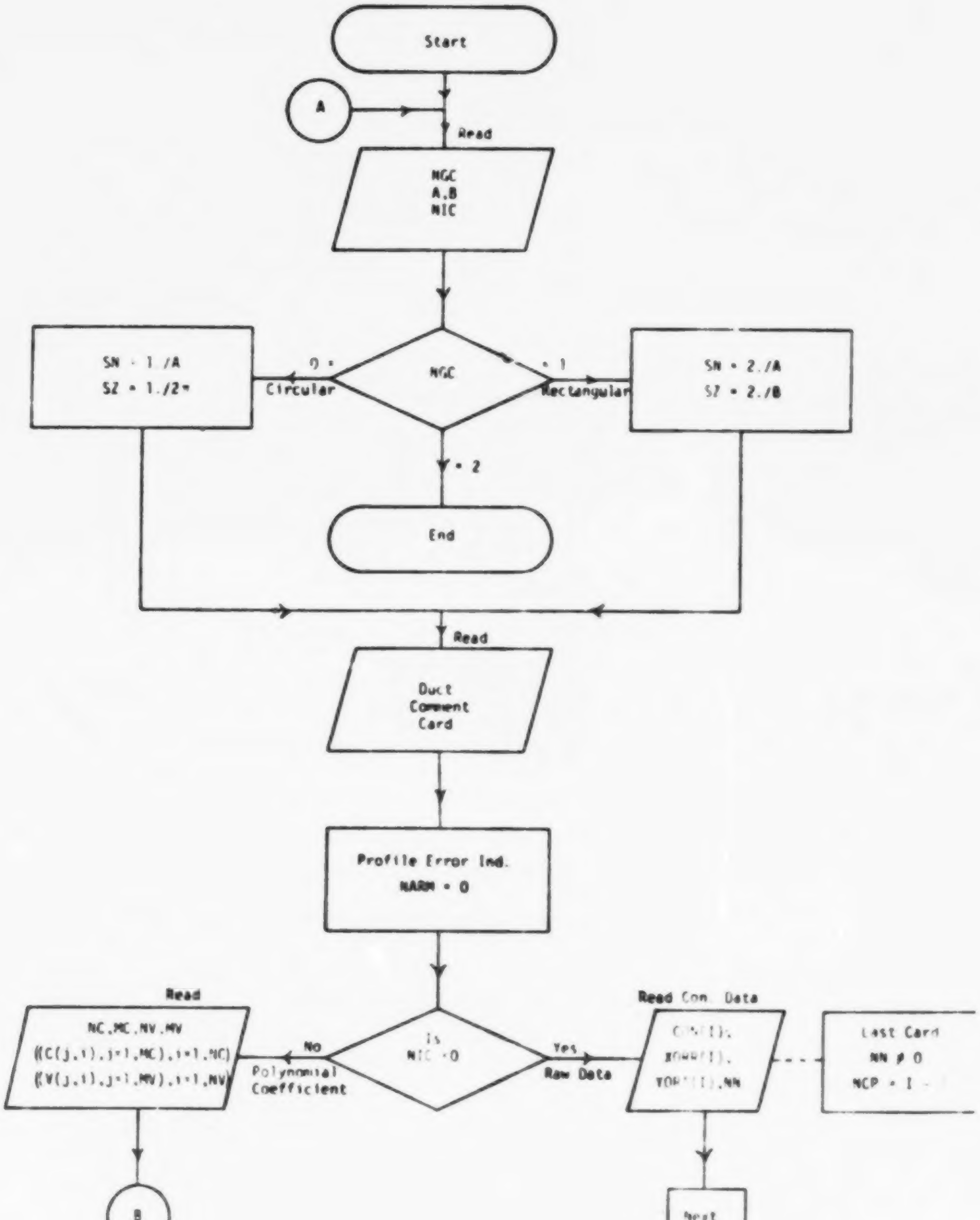
NALARM: Error indicator for the probe assignment data
  NALARM = 0   no error
  NALARM = 1   error

EXHIBIT C

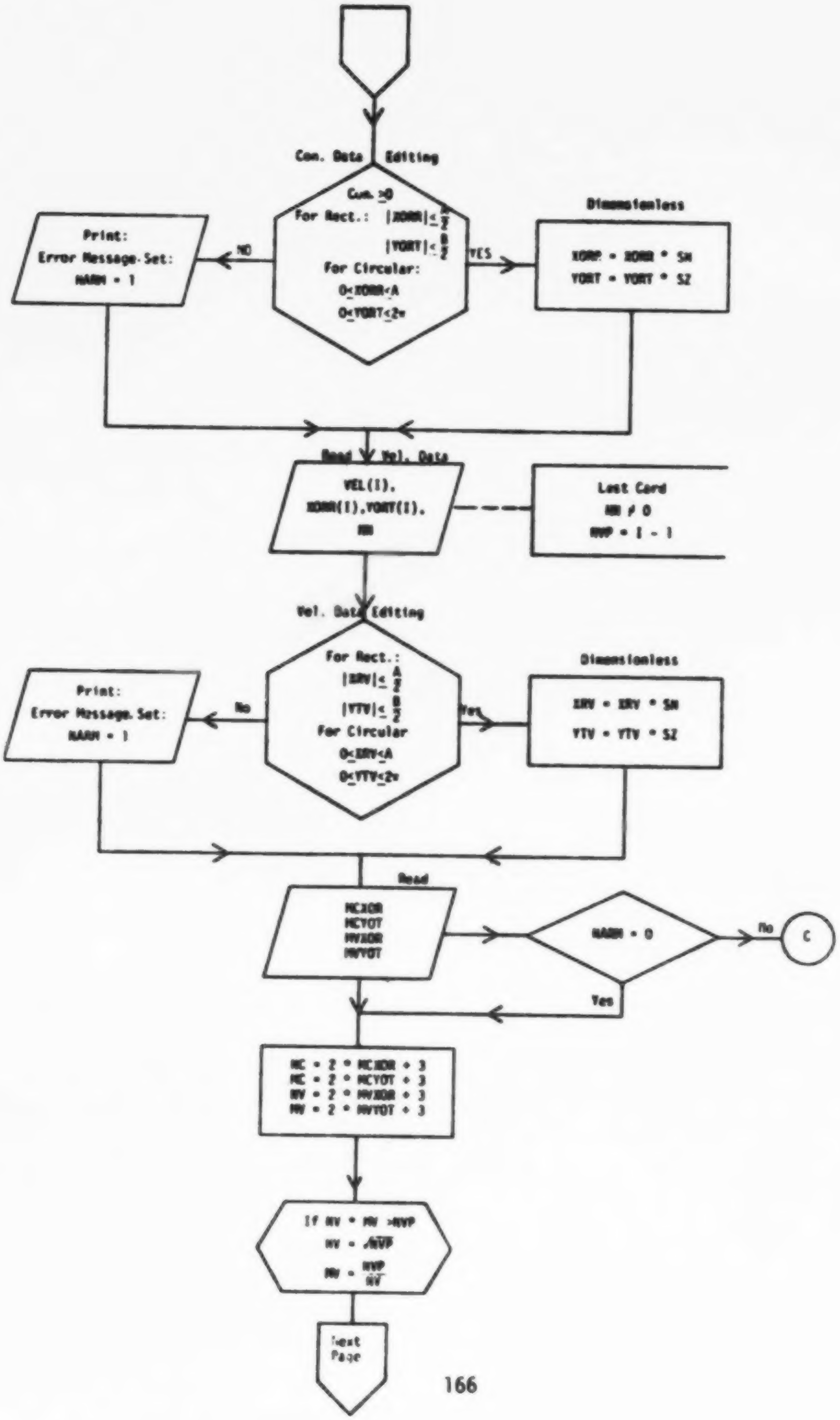
MAIN ROUTINE

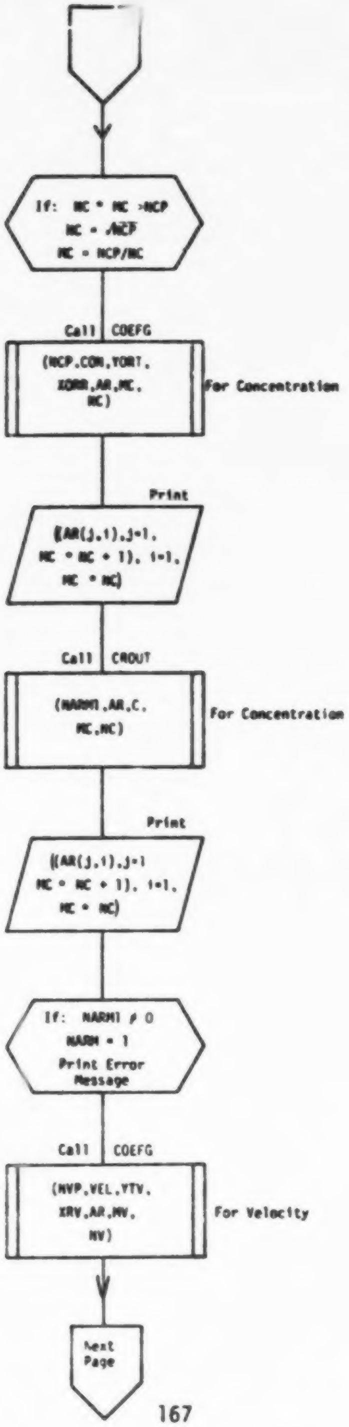
The main routine reads and prints the data and supervises the subsequent calling of the four subroutines and one function subprogram. For detailed information see Computational Procedures, Section II. Figure 6 is the main routine flow chart.

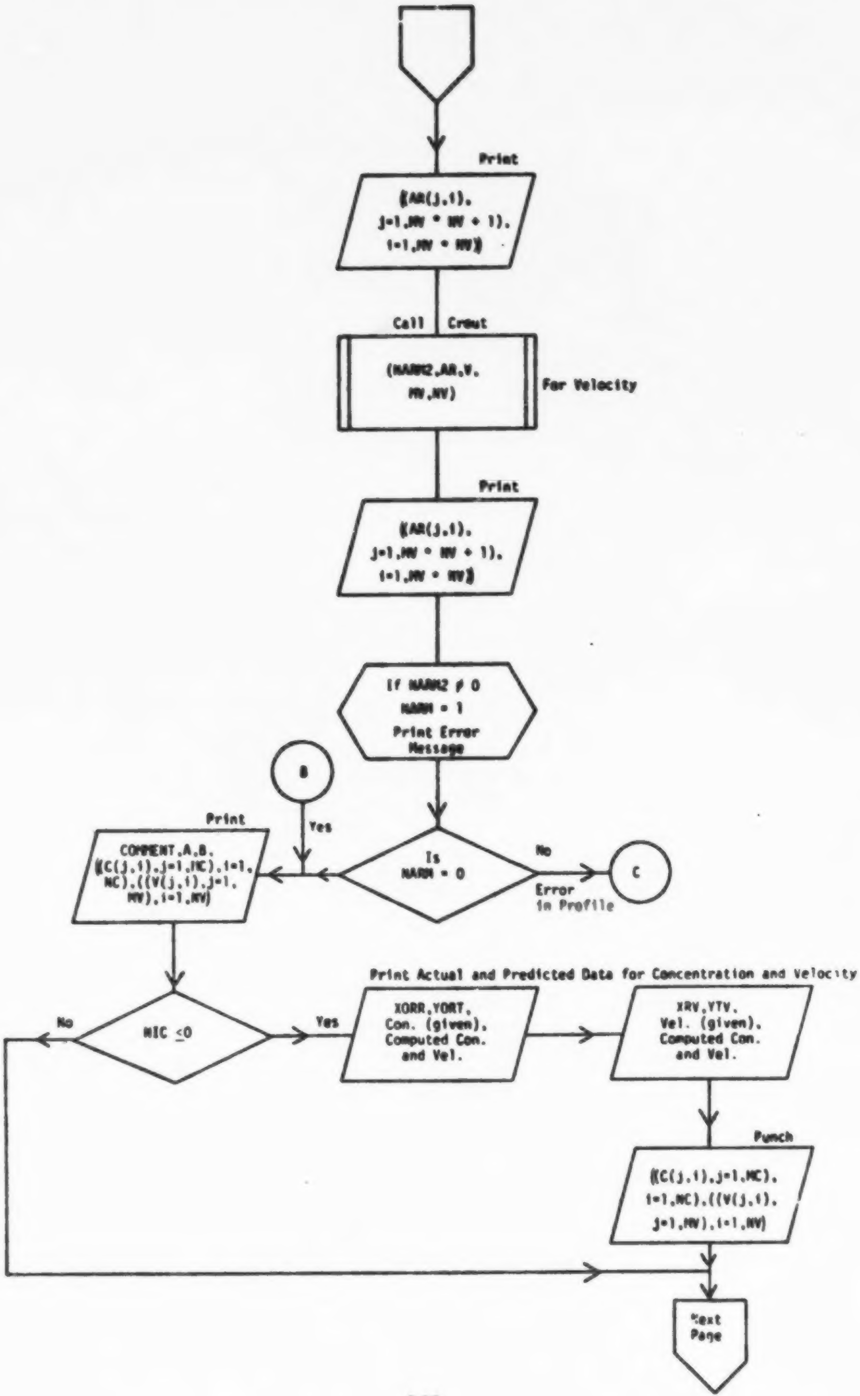
Figure 6. Program Flow Chart

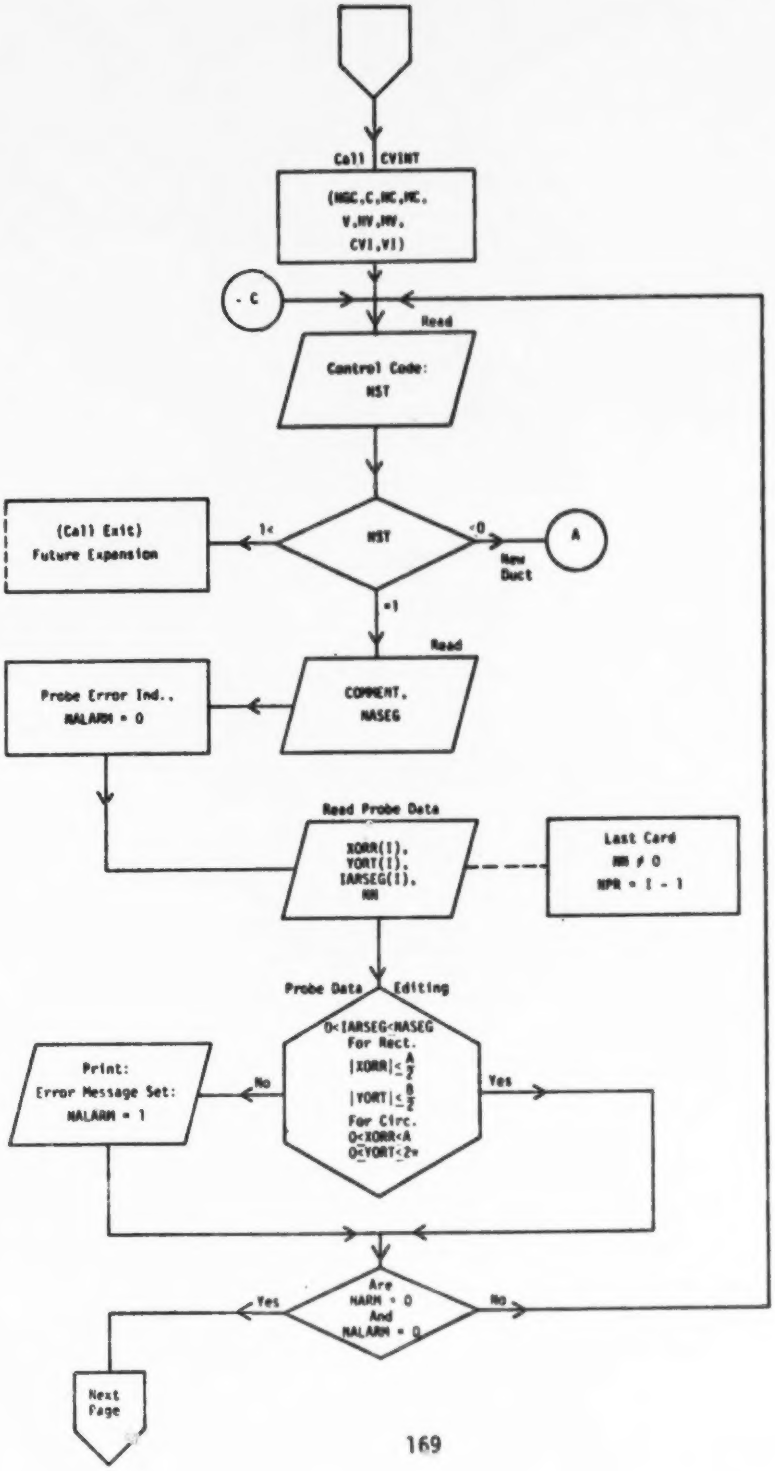


Start
A
Read
NGC
A,B
NIC
NGC
0 =
Circular
= 1
Rectangular
SN = 1./A
SZ = 1./2π
SN = 2./A
SZ = 2./B
= 2
End
Read
Duct
Comment
Card
Profile Error Ind.
NARM = 0
Read
NC,MC,NV,MV
((C(j,i),j=1,MC),i=1,NC)
((V(j,i),j=1,MV),i=1,NV)
No
Polynomial
Coefficient
Is
NIC = 0
Yes
Raw Data
Read Con. Data
CON(I),
XORR(I),
YORR(I),NN
Last Card
NN ≠ 0
NCP = I -
B
Next
Page


Con. Data Editing
Cum. ≥0
For Rect.: |XORR| ≤ A/2
|YORT| ≤ B/2
For Circular:
0≤XORR≤A
0≤YORT≤2π
NO
Print:
Error Message-Set:
NARM = 1
YES
Dimensionless
XORR = XORR * SN
YORT = YORT * SZ
Read Vel. Data
VEL(I),
XORR(I),YORT(I),
NN
Last Card
NN ≠ 0
NVP = I - 1
Vel. Data Editing
For Rect.:
|XRV| ≤ A/2
|YTV| ≤ B/2
For Circular
0≤XRV≤A
0≤YTV≤2π
No
Print:
Error Message.Set:
NARM = 1
Yes
Dimensionless
XRV = XRV * SN
YTV = YTV * SZ
Read
NCXOR
NCYOT
NVXOR
NVYOT
NARM = 0
No
C
Yes
NC = 2 * NCXOR + 3
MC = 2 * NCYOT + 3
NV = 2 * NVXOR + 3
MV = 2 * NVYOT + 3
If NV * MV > NVP
NV = √NVP
MV = NVP/NV
Next Page


If: NC * MC >NCP
NC = √NCP
MC = NCP/NC
Call COEFG
(NCP,CON,YORT,
XORR,AR,MC,
NC)
For Concentration
Print
((AR(j,i),j=1,
MC * NC + 1), i=1,
MC * NC)
Call CROUT
(NARM1,AR,C,
MC,NC)
For Concentration
Print
((AR(j,i),j=1
MC * NC + 1), i=1,
MC * NC)
If: NARM1 ≠ 0
NARM = 1
Print Error
Message
Call COEFG
(NVP,VEL,YTV,
XRV,AR,MV,
NV)
For Velocity
Next
Page


Print
((AR(j,i),
j=1,MV * NV + 1),
i=1,MV * NV)
Call Crout
(NARM2,AR,V,
MV,NV)
For Velocity
Print
((AR(j,i),
j=1,MV * NV + 1),
i=1,MV * NV)
If NARM2 ≠ 0
NARM = 1
Print Error
Message
B
Yes
Print
COMMENT,A,B,
((C(j,i),j=1,MC),i=1,
NC),((V(j,i),j=1,
MV),i=1,NV)
Is
NARM = 0
No
Error
in Profile
C
Print Actual and Predicted Data for Concentration and Velocity
No
NIC ≤0
Yes
XORR,YORT,
Con. (given),
Computed Con.
and Vel.
XRV,YTV,
Vel. (given),
Computed Con.
and Vel.
Punch
((C(j,i),j=1,MC),
i=1,NC),((V(j,i),
j=1,MV),i=1,NV)
Next
Page


Call CVINT
(NGC,C,NC,MC,
V,NV,MV,
CVI,VI)
C
Read
Control Code:
NST
(Call Exit)
Future Expansion
1<
NST
<0
New
Duct
A
=1
Read
COMMENT,
NASEG
Probe Error Ind.,
NALARM = 0
Read Probe Data
XORR(I),
YORT(I),
IARSEG(I),
NN
Last Card
NN ≠ 0
NPR = I - 1
Probe Data Editing
0<IARSEG<NASEG
For Rect.
|XORR|≤A/2
|YORT|≤B/2
For Circ.
0<XORR<A
0≤YORT≤2π
No
Yes
Print:
Error Message Set:
NALARM = 1
Are
NARM = 0
And
NALARM = 0
Yes
No
Next
Page

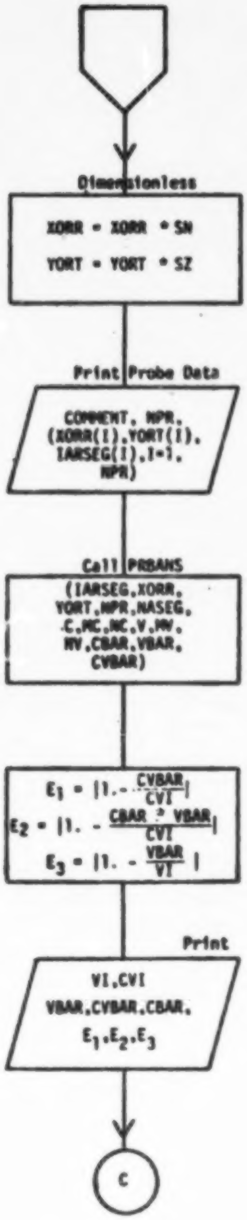

Dimensionless
XORR = XORR * SN
YORT = YORT * SZ
Print Probe Data
COMMENT, NPR, (XORR(I), YORT(I), IARSEG(I), I=1, NPR)
Call PRBANS
(IARSEG, XORR, YORT, NPR, NASEG, C, MC, NC, V, MV, NV, CBAR, VBAR, CVBAR)
$E_1 = |1. - \frac{CVBAR}{CVI}|$
$E_2 = |1. - \frac{CBAR * VBAR}{CVI}|$
$E_3 = |1. - \frac{VBAR}{VI}|$
Print
VI, CVI
VBAR, CVBAR, CBAR,
E_1, E_2, E_3
C

EXHIBIT D

SUBROUTINE: COEFG

A. PURPOSE:

The least square curve fitting of the polynomial,

$$Z = \sum_{i=1}^{N} \sum_{j=1}^{M} a_{ji} y^{j-1} x^{i-1}$$

results in a set of (N*M) linear equations with (N*M) unknowns. The purpose of this routine is to generate and store the coefficients of that set of equations in an array.

B. USAGE: CALL COEFG(NP, Z, Y, X, AR, M, N)

C. DESCRIPTION OF PARAMETERS:

1. INPUT TO THE ROUTINE:

NP: Number of data points

Z: Dependent variable data, size: Z (NP)

y, x: Independent variable data, size: x(NP), y(NP)

M-1: Order of polynomial in y

N-1: Order of polynomial in x

2. OUTPUT FROM THE ROUTINE

AR: Matrix of coefficients of equations, Size: AR(M*N, M+N +1)

D. REMARKS: $0<(N,M)\leq 11$

$M*N\leq NP\leq 300$

E. SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED: None

F. DESCRIPTION AND METHOD:

Least squares polynomial curve fitting with two independent variables:

$$Z = \sum_{i=1}^{N} \sum_{j=1}^{M} a_{ji} y^{j-1} x^{i-1}$$

$$M = \sum_{k=1}^{NP} (Z_k^{'}-Z_k)^2$$

where $Z_k^{'}$ is the predicted value and Z_k is the actual value of point k

for least squares:

$$\frac{\partial M}{\partial a_{ji}} = 0 \qquad \begin{cases} 1\leq i\leq N \\ 1\leq j\leq M \end{cases}$$

$$\therefore \sum_{k=1}^{NP} 2(Z_k^{'}-Z_k)\, y_k^{j-1} x_k^{i-1} = 0 \qquad \begin{cases} 1\leq i\leq N \\ 1\leq j\leq M \end{cases}$$

or

$$\sum_{k=1}^{NP} Z_k^{'} y_k^{j-1} x_k^{i-1} = \sum_{k=1}^{NP} Z_k y_k^{j-1} x_k^{i-1}$$

since:

$$Z_k^{'} = \sum_{s=1}^{N} \sum_{r=1}^{M} a_{rs} y_k^{r-1} x_k^{s-1}$$

$$\therefore \sum_{k=1}^{NP} Z_k^{'} y_k^{j-1} x_k^{i-1} = \sum_{k=1}^{NP} \left(\sum_{s=1}^{N} \sum_{r=1}^{M} a_{rs} y_k^{r-1} x_k^{s-1} \right) y_k^{j-1} x_k^{i-1}$$

$$= \sum_{k=1}^{NP} Z_k y_k^{j-1} x_k^{i-1}$$

or

$$\sum_{s=1}^{N} \sum_{r=1}^{M} \left(\sum_{k=1}^{NP} y_k^{r+j-2} x_k^{s+i-2} \right) a_{rs} = \sum_{k=1}^{NP} Z_k y_k^{j-1} x_k^{i-1} \qquad \left\{ \begin{array}{l} 1 \leq i \leq n \\ 1 \leq j \leq M \end{array} \right.$$

Thus, providing NP$\geq$N*M, a set of (N*M) linear equations with N*M unknowns, which are linearly independent are found. The function of COEFG routine is to find the coefficients of these equations and store them in matrix AR (M*N, M*N+1). The unknowns (a_{rs}, coefficients of the polynomial) are found by CROUT-JORDAN REDUCTION METHOD in CROUT ROUTINE.

EXHIBIT E

SUBROUTINE: CROUT

A. PURPOSE: To solve M*N linear equations with M*N unknowns, which are linearly independent.

B. USAGE:

Call CROUT (NALARM, AR, CORV, M, N)

C. DESCRIPTION OF PARAMETERS:

1. INPUT TO THE ROUTINE:

AR: Matrix of coefficients (output of COEFG). This array is rearranged during execution with the last column becoming the solution of equations.

Size: AR (M*N, M*N + 1)

M-1: Order of polynomial in y or Θ

N-1: Order of polynominal in x or R

2. OUTPUT FROM THE ROUTINE:

CORV: Matrix of coefficients of the polynomial,

$$\sum_{i=1}^{N} \sum_{j=1}^{M} \mathrm{CORV}(j,i) Y^{j-1} X^{i-1}$$

Size: CORV(M,N)

NALARM: 0 if the routine is able to solve the problem.
1 if the routine is unable to solve the problem.
(very rare, unavoidable division by zero or small numbers).

D. REMARKS: $0<(M,N)\leq 11$

E. SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED: None

F. DESCRIPTION AND METHOD:

The input coefficients whose absolute values are less than TOL = 10^{-9} are zerorized, then the CROUT-JORDAN REDUCTION METHOD (see Appendix I) is used to solve the N*M equations with N*M unknowns. In addition raw transformation is performed, wherever necessary, to avoid division by zero or small numbers (less than TOL). The solution of the equations (the last column of AR matrix) is stored in polynomial coefficients matrix, CORV(M,N).

EXHIBIT F

SUBROUTINE: CVINT

A. PURPOSE:

Obtain average velocity and average emission from the given velocity and concentration polynomials.

B. USAGE:

CALL CVINT(NGC, C, NC, MC, V, NV, MV, CVI, VI)

C. DESCRIPTION OF PARAMETERS:

1. INPUT TO THE ROUTINE:

NGC: Geometry code; 0 circular duct.
1 rectangular duct.

C: Matrix whose elements are coefficients of the concentration polynomial;

$$\text{Concentration} = \sum_{i=1}^{NC} \sum_{j=1}^{MC} C_{ji} y^{j-1} x^{i-1} \quad \text{(for rectangular)}$$

or

$$\text{Concentration} = \sum_{i=1}^{NC} \sum_{j=1}^{MC} C_{ji} \theta^{j-1} R^{i-1} \quad \text{(for circular)}$$

where (y,x) or θ,R are dimensionless variables;

$$x = \frac{x_{in}}{A/2},\ y = \frac{y_{in}}{B/2},\ R = \frac{R_{in}}{A},\ \text{and } \theta = \frac{\theta_{in}}{2\pi}. \qquad \text{Size: } C(MC, NC)$$

NC: ORDER + 1 of concentration polynomial in x or R

MC: ORDER + 1 of concentration polynomial in y or theta

V: Matrix of velocity polynomial coefficients similar to C
Size: V(MV, NV)

NV: ORDER + 1 of velocity polynomial in x or R

MV: ORDER + 1 of velocity polynomial in y or theta

2. OUTPUT FROM THE ROUTINE:

CVI: average emission per unit time and per unit area

VI: average velocity

D. REMARKS: $0<(NC, MC, NV, MV)\leq 11$

E. SUBROUTINE AND FUNCTION SUBPROGRAMS REQUIRED: None

F. DESCRIPTION AND METHOD:

Given velocity and concentration polynomials:

For rectangular duct:

$$V(x,y) = \sum_{i=1}^{NV} \sum_{j=1}^{MV} V_{ji} y^{j-1} x^{i-1}$$

$$C(x,y) = \sum_{i=1}^{NC} \sum_{j=1}^{MC} C_{ji} y^{j-1} x^{i-1}$$

where x and y are dimensionless $\left(\frac{x_{in}}{A/2}, \frac{y_{in}}{B/2}\right)$

For circular duct:

$$V(R,\theta) = \sum_{i=1}^{NV} \sum_{j=1}^{MV} V_{ji}\theta^{j-1}R^{i-1}$$

$$C(R,\theta) = \sum_{i=1}^{NC} \sum_{j=1}^{MC} C_{ji}\theta^{j-1}R^{i-1}$$

and;

$$VI = \frac{\int_A VdA}{A} \qquad CVI = \frac{\int_A VCdA}{A} \qquad \text{where } A = \text{area}$$

Thus, for Rectangular Duct:

$$VI = \int_{-1}^{1}\int_{-1}^{1} V(x,y)dxdy/2*2$$

$$= \int_{-1}^{1}\int_{-1}^{1} \sum_{i=1}^{NV} \sum_{j=1}^{MV} V_{ji}y^{j-1}x^{i-1}\,dxdy/4$$

$$VI = \left| \sum_{i=1}^{NV} \sum_{j=1}^{MV} \frac{V_{ji}}{4ji}\, y^j x^i \right|_{-1}^{1}\Bigg|_{-1}^{1}$$

$$\boxed{VI = \sum_{i=1}^{NV} \sum_{j=1}^{MV} \frac{V_{ji} \left|1-(-1)^{j}\right| \left|1-(-1)^{i}\right|}{4ji}}$$

Note: if i and j both be odd $\left|1-(-1)^{j}\right| \left|1-(-1)^{i}\right| = 4$

otherwise $= 0$

$$CVI = \int_{-1}^{1}\int_{-1}^{1} C(x,y)V(x,y)dxdy/4$$

$$= \frac{1}{4}\int_{-1}^{1}\int_{-1}^{1} \left(\sum_{i=1}^{NC} \sum_{j=1}^{MC} C_{ji} y^{j-1} x^{i-1} \right) \left(\sum_{k=1}^{NV} \sum_{\ell=1}^{MV} V_{\ell k} y^{\ell-1} x^{k-1} \right) dxdy$$

$$\boxed{CVI = \sum_{i=1}^{NC} \sum_{j=1}^{MC} \sum_{k=1}^{NV} \sum_{\ell=1}^{MV} \frac{C_{ji} V_{\ell k} \left[1-(-1)^{j+\ell-1}\right] \left[1-(-1)^{i+k-1}\right]}{4(j+\ell-1)(i+k-1)}}$$

Note: if $(j+\ell-1)$ and $(i+k-1)$ both are odd;

$\left|1-(-1)^{j+\ell-1}\right| \left|1-(-1)^{i+k-1}\right| = 4$

otherwise $= 0$

For Circular Duct:

$$VI = \int_0^{2\pi} \int_0^1 V(R,\theta)RdRd\theta'/\pi(1)^2 \qquad \text{where } \theta' = 2\pi\theta$$

$$= \int_0^1 \int_0^1 2V(R,\theta)RdRd\theta$$

$$= \int_0^1 \int_0^1 2\sum_{i=1}^{NV} \sum_{j=1}^{MV} V_{ji}\theta^{j-1}\ {}^{i-1}\ d\ d\theta$$

$$\boxed{VI = \sum_{i=1}^{NV} \sum_{j=1}^{MV} \frac{2V_{ji}}{j(i+1)}}$$

and

$$CVI = \int_0^1 \int_0^1 2C(R,\theta)V(R,\theta)RdRd\theta$$

$$= \int_0^1 \int_0^1 2\sum_{i=1}^{NC} \sum_{j=1}^{MC} \sum_{k=1}^{NV} \sum_{\ell=1}^{MV} C_{ji}V_{\ell k}\theta^{j+\ell-2}R^{i+k-1}dRd\theta$$

$$\boxed{CVI = \sum_{i=1}^{NC} \sum_{j=1}^{MC} \sum_{k=1}^{NV} \sum_{\ell=1}^{MV} \frac{2C_{ji}V_{1k}}{(j+\ell-1)(i+k)}}$$

EXHIBIT G

SUBROUTINE: PRBANS

A. PURPOSE

Given velocity and concentration polynominals and probes locations and equal area segment numbers, find average concentration, velocity and emission based on equal area strategy.

B. USAGE

CALL PRBANS (IARSEG, XORR, YORT, NPR, NASEG, C, MC, NC,
V, MV, NV, CBAR, VBAR, CVBAR)

C. DESCRIPTION OF PARAMETERS

1. Input to the Routine

IARSEG(I): No. of area segments where probe I is located. Size: IARSEG(NPR)

XORR(I): X or R of probe I Size: XORR(NPR)

YORT(I): Y or THETA of probe I Size: YORT(NPR)

NPR: No. of probes

NASEG: Total no. of area segments

C: Matrix of coefficients of concentration polynominal. Size: C (MC, NC)

MC: Order + 1 of concentration polynominal in Y or THETA

NC: Order + 1 of concentration polynominal in X or R.

V: Matrix of coefficients of velocity polynominal. Size: V (MV, NV)

MV: Order + 1 of velocity polynominal in Y or THETA.

NV: Order + 1 of velocity polynominal in X or R.

2. Output from the Routine

CBAR: Average concentration.

VBAR: Average velocity.

CVBAR: Average emission.

D. REMARKS

$$0 < (NPR, NASEG) \leq 300$$

$$0 < (NC, MC, NV, MV) \leq 11$$

E. SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED:

POLGEN (Function).

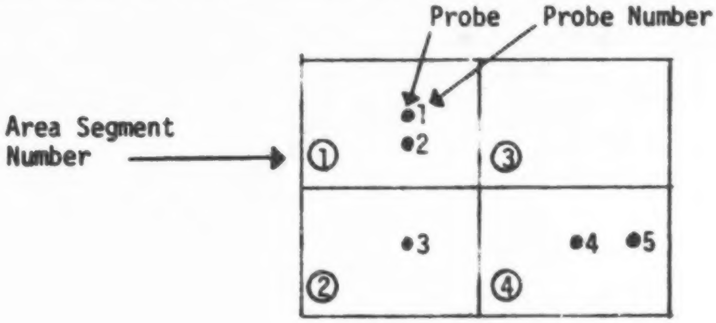
F. DESCRIPTION AND METHOD

$$\overline{C} = \frac{\sum_{i=1}^{NPR} \frac{C_i}{IUSEG\ (IARSEG(I))}}{\text{No. of area segments with probe (ICZ)}}$$

$$\overline{V} = \frac{\sum_{i=1}^{NPR} \frac{V_i}{IUSEG\ (IARSEG(I))}}{ICZ}$$

$$\overline{CV} = \frac{\sum_{i=1}^{NPR} \frac{C_i \, V_i}{IUSEG\ (IARSEG(I))}}{ICZ}$$

where IUSEG(N) is the number of probes in area segment N. Example:



NPR = 5

NASEG = 4

IARSEG (1) = 1
IARSEG (2) = 1
IARSEG (3) = 2
IARSEG (4) = 4
IARSEG (5) = 4

It means that probes 1, 2, 3, 4 and 5 are respectively located in area segment 1, 1, 2, 4 and 4.

IUSEG (1) = 2
IUSEG (2) = 1
IUSEG (3) = 0
IUSEG (4) = 2

It means that there are 2 probes in area segment 1, 1 probe in area segment 2, 0 probes in area segment 3, and 2 probes in area segment 4.

ICZ = 3

There are 3 area segments with probe.

$$\overline{C} = \frac{(\frac{C1}{2} + \frac{C2}{2}) + (\frac{C3}{1}) + (\frac{C4}{2} + \frac{C5}{2})}{3}$$

and similarly for $\overline{V}$ and $\overline{CV}$.

EXHIBIT H

FUNCTION POLGEN

A. PURPOSE

Evaluates the polynominal,

$$\sum_{i=1}^{N} \sum_{j=1}^{M} (\text{CORV})_{ji}\, Y^{j-1}\, x^{i-1}$$

at a given x and Y.

B. USAGE

A = POLGEN (CORV, M, N, X, Y)

C. DESCRIPTION OF PARAMETERS

CORV: Matrion of polynominal coefficients.
Size: CORV (M, N)

M: Order + 1 of polynominal in Y

N: Order + 1 of polynominal in X

X,Y: Location where the polynominal is desired to be evaluated.

D. REMARKS

$$0 < (M, N) \leq 11$$

E. DESCRIPTION AND METHOD

$$\text{POLGEN} = \sum_{i=1}^{N} \sum_{j=1}^{M} (\text{CORV})_{ji}\, Y^{j-1}\, x^{i-1}$$

EXHIBIT I

The Crout Method for Solving Sets of Linear Algebraic Equations

The method of Crout, for solving a set of n linear algebraic equations in n unknowns, is basically equivalent to the method of Gauss (see footnote to page 4). However, the calculations are systematized in such a way that they are conveniently carried out on a desk calculator, with a minimum number of separate machine operations. Furthermore, a very considerable saving of time and labor results from the fact that the recording of auxiliary data is minimized and compactly arranged.

Only a description and illustration of the method is given here; an analytic justification is included in the original paper.*

The calculation proceeds from the *augmented matrix* of the system,

$$\mathbf{M} = \left[\begin{array}{cccc:c} a_{11} & a_{12} & \cdots & a_{1n} & c_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & c_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} & c_n \end{array}\right] = [\mathbf{a}\,\vdots\,\mathbf{c}], \tag{1}$$

which may be considered as partitioned into the coefficient matrix **a** and the column vector **c**, to an *auxiliary matrix*

$$\mathbf{M}' = \left[\begin{array}{cccc:c} a'_{11} & a'_{12} & \cdots & a'_{1n} & c'_1 \\ a'_{21} & a'_{22} & \cdots & a'_{2n} & c'_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a'_{n1} & a'_{n2} & \cdots & a'_{nn} & c'_n \end{array}\right] = [\mathbf{a}'\,\vdots\,\mathbf{c}'], \tag{2}$$

of the same dimensions, and thence to the required *solution vector*

$$\mathbf{x} = \begin{Bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{Bmatrix}. \tag{3}$$

It is convenient to define the *diagonal element* of any element to the *right* of the principal diagonal of a matrix as that element of the principal diagonal which lies in the same *row* as the given element. The diagonal element of any element *below* the principal diagonal is defined as that element of the principal diagonal which lies in the same *column* as the given element.

---

* See Reference 7 to Chapter 1.

With this definition, the procedure for obtaining the elements of **M**′ from those of the given matrix **M** may be described by the four rules which follow:

1. The elements of **M**′ are determined in the following order: elements of the first column, then elements of the first row to the right of the first column; elements of the second column below the first row, then elements of the second row to the right of the second column; and so on, until all elements are determined.

2. The first column of **M**′ is identical with the first column of **M**. Each element of the first row of **M**′ except the first is obtained by dividing the corresponding element of **M** by the leading element a_{11}.

3. Each element a'_{ij} on or below the principal diagonal of **M**′ is obtained by subtracting from the corresponding element a_{ij} of **M** the sum of the products of elements in the ith row and corresponding elements in the jth column of **M**′, all uncalculated elements being imagined to be zeros. In symbols, we thus have

$$a'_{ij} = a_{ij} - \sum_{k=1}^{j-1} a'_{ik}a'_{kj} \qquad (i \geqq j). \tag{4}$$

4. Each element a'_{ij} to the right of the principal diagonal is calculated by the procedure of Rule 3 *followed by a division by the diagonal element* a'_{ii} in **M**′. Thus there follows

$$a'_{ij} = \frac{a_{ij} - \sum_{k=1}^{i-1} a'_{ik}a'_{kj}}{a'_{ii}} \qquad (i < j). \tag{5}$$

In the important cases when the coefficient matrix **a** is *symmetric* $(a_{ji} = a_{ij})$, it can be shown that *any element* a'_{ij} *to the right of the principal diagonal is equal to the result of dividing the symmetrically placed element* a'_{ji} (below the diagonal) *by its diagonal element* a'_{ii}. This fact reduces the labor involved in the formation of **M**′, in such a case, by a factor of nearly two, since each element *below* the diagonal may be recorded as a by-product of the calculation of the symmetrically placed element, before the final division is effected.

The procedure for obtaining the final solution vector **x** from the matrix **a**′ and the vector **c**′, into which **M**′ is partitioned, may be described by the three rules which follow:

1. The elements of **x** are determined in the reverse order $x_n, x_{n-1}, x_{n-2}, \ldots, x_1$, from the last element to the first.

2. The last element x_n is identical with the last element c'_n of **c**′.

3. Each element x_i of **x** is obtained by subtracting from the corresponding element c'_i of **c'** the sum of the products of elements in the ith row of **a'** by corresponding elements of the column **x**, all uncalculated elements of **x** being imagined to be zeros. Thus there follows

$$x_i = c'_i - \sum_{k=i+1}^{n} a'_{ik}x_k. \tag{6}$$

The solution may, of course, be checked completely by substitution into the n basic linear equations. However, if desired, a *check column* may be carried along in the calculation to provide a *continuous* check on the work. Each element of the initial check column, corresponding to the augmented matrix **M**, is the sum of the elements of the corresponding row of **M**. If this column is recorded to the right of the augmented matrix, *and is treated in the same manner as the column* **c**, corresponding check columns are thus obtained for the auxiliary matrix **M'**, and for the solution vector **x**. Continuous checks on the calculation are then afforded by the two rules which follow:

1. In the auxiliary matrix, any element of the check column should *exceed by unity* the sum of the other elements in its row which lie *to the right of the principal diagonal.*

2. Each element of the check column associated with the solution vector should *exceed by unity* the corresponding element of the solution vector.

The preceding checks serve, not only to display numerical errors, but also to give an estimate of the effect of round-off errors resulting from the retention of insufficiently many significant figures.*

The simplicity and efficiency of this procedure can be appreciated only when it is applied to specific problems and compared with other procedures.

It is of some interest to notice that (as is shown in the reference cited) the set of equations which would be obtained by the direct use of the Gauss reduction would possess the augmented matrix

$$\tilde{\mathbf{M}} = \left[\begin{array}{cccc:c} 1 & a'_{12} & \cdots & a'_{1n} & c'_1 \\ 0 & 1 & \cdots & a'_{2n} & c'_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 & c'_n \end{array}\right] = [\tilde{\mathbf{a}} \mathbin{\vdots} \mathbf{c}'], \tag{7}$$

which differs from (2) only in the substitution of ones in the principal diagonal and zeros below it. The transition from the set of equations represented by (7) to the solution (6) is seen to correspond to the "back solution" of the Gauss procedure. *The compactness of the Crout procedure*

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* Appreciable loss of accuracy may be encountered if a *small diagonal element* appears in **M'** at an early stage of the calculation. Such a situation may often be remedied by reordering the equations and/or unknowns.

is achieved by recording auxiliary data in the spaces of $\mathfrak{M}$ which would otherwise be occupied by ones and zeros.

The ith diagonal element of $\mathbf{a}'$ happens to be the coefficient by which the ith equation is divided, before that equation is used to eliminate the ith unknown from succeeding equations in the Gauss reduction. Since all other steps in the reduction do not affect the *determinant* of the coefficient matrix, it follows that *the determinant of* $\mathbf{a}$ *is equal to the product of the diagonal elements of* $\mathbf{a}'$,

$$|\mathbf{a}| = a'_{11}a'_{22} \cdots a'_{nn}. \tag{8}$$

Thus the Crout procedure is useful also in evaluating determinants, the columns $\mathbf{c}$ and $\mathbf{c}'$, as well as the final vector $\mathbf{x}$, then being omitted.

In the reference cited, it is shown that the method can be extended to the convenient treatment of equations with *complex* coefficients, and to the case of m equations in n unknowns.

In order to illustrate the procedure numerically, we apply it to the system

$$\begin{aligned} 554.11\,x_1 - 281.91\,x_2 - 34.240x_3 &= 273.02, \\ -281.91\,x_1 + 226.81\,x_2 + 38.100x_3 &= -63.965, \\ -34.240x_1 + 38.100x_2 + 80.221x_3 &= 34.717, \end{aligned}$$

with the *augmented matrix* (and associated check column)

$$\begin{array}{cc} & \textit{Check} \\ \mathbf{M} = \begin{bmatrix} \underline{554.11} & -281.91 & -34.240 & 273.02 \\ -281.91 & \underline{226.81} & 38.100 & -63.965 \\ -34.240 & 38.100 & \underline{80.221} & 34.717 \end{bmatrix} \cdot & \begin{matrix} 510.98 \\ -80.965 \\ 118.80 \end{matrix} \end{array}$$

The *auxiliary matrix* (and associated check column) are obtained in the form

$$\begin{array}{cc} & \textit{Check} \\ \mathbf{M}' = \begin{bmatrix} \underline{554.11} & -0.50876 & -0.061793 & 0.49272 \\ -281.91 & \underline{83.385} & 0.24801 & 0.89870 \\ -34.240 & 20.680 & \underline{72.976} & 0.45224 \end{bmatrix} \cdot & \begin{matrix} 0.92216 \\ 2.14668 \\ 1.45228 \end{matrix} \end{array}$$

and the *solution vector* (and final check column) are found to be

$$\begin{array}{cc} & \textit{Check} \\ \mathbf{x} = \begin{Bmatrix} 0.92083 \\ 0.78654 \\ 0.45224 \end{Bmatrix}, & \begin{matrix} 1.92080 \\ 1.78650\checkmark \\ 1.45228 \end{matrix} \end{array}$$

if all calculated values are rounded off to five significant figures throughout the calculation. Thus there follows

$$x_1 = 0.92083, \quad x_2 = 0.78654, \quad x_3 = 0.45224,$$

where reference to the final check column indicates the possible presence of round-off errors of the order of four units in the last place retained. Such errors would be decreased by retaining additional significant figures in the formation of **M**′ and **x**.

The elements of the first column of **M**′ are identical with the corresponding elements of **M**; the elements of the first row of **M**′ following the first element are obtained by dividing the corresponding elements of **M** by 554.11. The remaining elements of **M**′ are determined as follows:

$$a'_{22} = 226.81 - (-281.91)(-0.50876) = 83.385.$$

$$a'_{32} = 38.100 - (-0.50876)(-34.240) = 20.680.$$

$$a'_{23} = \frac{38.100 - (-0.061793)(-281.91)}{83.385} = \frac{20.680}{83.385} = 0.24801.$$

$$c'_2 = \frac{-63.965 - (0.49272)(-281.91)}{83.385} = 0.89870.$$

$$a'_{33} = 80.221 - (-0.061793)(-34.240) - (0.24801)(20.680) = 72.976.$$

$$c'_3 = \frac{34.717 - (0.49272)(-34.240) - (0.89870)(20.680)}{72.976} = 0.45224.$$

The last element of **x** is identical with c'_3. The remaining elements of **x** are determined as follows:

$$x_2 = 0.89870 - (0.24801)(0.45224) = 0.78654.$$

$$x_1 = 0.49272 - (-0.50876)(0.78654) - (-0.061793)(0.45224) = 0.92083.$$

It may be noticed that, because of the symmetry of the coefficient matrix, it is not actually necessary to calculate a'_{23} and a'_{32} independently. If a'_{23} is calculated, the numerator (20.680) may be recorded as a'_{32} before the final division is effected.

The advantages of the procedure (and the additional simplifications introduced by symmetry of the matrix **a**) increase with the number of equations involved.

It is particularly important to notice that the calculation of each element of either **M**′ or **x** involves only a *single continuous machine operation* (a sum of products, with or without a final division), without the necessity of intermediate tabulation or transfer of auxiliary data.

If the *determinant* of the coefficient matrix were required, it would be obtained as the product of the diagonal elements of **a**′:

$$|\mathbf{a}| = (554.11)(83.385)(72.976) = 3.3718 \times 10^7.$$

EXHIBIT J

SOURCE LISTING

```
C     MAIN PROGRAM
C     THIS PROGRAM COMPUTES THE ERROR ASSOCIATED WITH COMPUTING TOTAL
C     STACK GAS EFFLUENT AND SPECIFIC SPECIES EFFLUENT FOR SPECIFIC
C     ALLOCATION OF VELOCITY AND CONCENTRATION PROBES IN CIRCULAR OR
C     RECTANGULAR DUCTS WHERE THE AREA ALLOCATION STRATEGY IS SPECIFIED
C     AND THE ACTUAL VELOCITY AND CONCENTRATION PROFILES IN THE DUCT ARE
C     SPECIFIED
C                     MEANING OF INPUT  CODES
C     CODE  VALUE  MEANING
C   1.NGC     0     CIRCULAR CROSS SECTIONED DUCT
C             1     RECTANGULAR CROSS SECTIONED DUCT
C   2.NIC     0     RAW DATA POINTS SUBMITTED FOR ACTUAL DUCT PROFILES
C             1     POLYNOMIAL COEFFICIENTS SUBMITTED FOR DUCT PROFILES
C   3.NCXOR <=4     NUMBER OF MAXIMA AND MINIMUM POINTS IN CONC. PROFILE
C                   IN X IF RECTANGULAR DUCT OR R IF CIRCULAR DUCT
C   4.NCYOT <=4     NUMBER OF MAXIMA AND MINIMUM POINTS IN CONC. PROFILE
C                   IN Y IF RECTANGULAR DUCT OR THETA IF CIRCULAR DUCT
C   5.NVXOR <=4     NUMBER OF MAXIMA AND MINIMA IN PROFILE OF VELOCITY
C                   IN X IF RECTANGULAR OR R IF CIRCULAR DUCT
C   6.NVYOT <=4     NUMBER OF MAXIMA AND MINIMA
C                   IN Y IF RECTANGULAR OR THETA IF CIRCULAR DUCT
C              NOTE ITEMS 3-6 ARE SPECIFIED ONLY IF NIC = 0
C   7.NST     0     ANOTHER DUCT (NEW DUCT)
C             1     EQUAL AREA STRATEGY CODE
C             2
C             3
C             4
C             5
C   8.NASEG         NUMBER OF EQUAL AREA SEGMENTS
C           OTHER INPUT DATA
C     IF NGC=0  A - RADIUS OF CIRCLE
C          =1  A,B- SHORT AND LONG DIMENSIONS RECTANGLE
C     IF NCI=0 (CON(I),XORR(I),YORT(I),I=1,NCP) CONCENTRATION AND VELOC.
C              (VEL(I), XRV(I), YTV(I),I=1,NVP) AT X,Y OR R,THETA
C     IF NCI=1((C(J,I),J=1,MC),I=1,NC)       WHERE C(J,I) AND V(J,I)
C              ((V(J,I),J=1,MV),I=1,NV)        ARE THE COEFFICIENTS OF
C                                              THE CONCENTRATION AND
C                                              VELOCITY PROFILES RESPEC.
C                                            MV,MC IS THE ORDER + 1 OF
C                                              POLYNOMIAL IN Y OR THETA
C                                              AND NV,NC IS THE ORDER OF
C                                              THE POLYNOMIAL IN X OR R.
C     (XORR(I),YORT(I),IARSEG(I),I=1,NPR)    PROBE LOCATION COORDINATES
C                                            AND AREA SEGMENT NUMBER
C
C        OTHER PARAMETERS
C     NCP             NUMBER OF CONCENTRATION DATA POINTS
C     NVP             NUMBER OF VELOCITY DATA POINTS
```

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```
               C      NPB          NUMBER OF PROBES.
               C      NASM   ERROR INDICATOR FOR DUCT PROFILES.
               C      NALARM  ERROR INDICATOR FOR PROBE ASSIGNMENT DATA.
 0001                 DIMENSION AR(121,122),CON(300),VEL(300),XORR(300),YORT(300),YTV(30
                     10),      C(11,11),V(11,11), IARSEG(300)          ,XRV(300)
 0002                 DIMENSION COMENT(20)
 0003               1 FORMAT(I1,1X,F12.5,2X,F12.5,1X,I1)
 0004               2 FORMAT(I3)
 0005               3 FORMAT(3F12.5,40X,I4)
 0006               4 FORMAT(4I2)
 0007               5 FORMAT(5F16.9)
 0008               6 FORMAT(I11)
 0009               7 FORMAT(2F12.5,1X,I3,48X,I4)
 0010               8 FORMAT(10X,'STACK GEOMETRY')
 0011               9 FORMAT(15X,'CIRCULAR-RADIUS=',F12.5,'FT.')
 0012              10 FORMAT(15X,'RECTANGULAR-A(FT.)=',F12.5,2X,'B(FT.)=',F12.5)
 0013              11 FORMAT(20X,'PROFILE DATA'/5X,'CONCENTRATION COEFFICIENTS')
 0014              12 FORMAT(1X,'C',I2,',',I2,'=',E16.9)
 0015              13 FORMAT(5X,'VELOCITY,COEFFICIENTS')
 0016              14 FORMAT(1X,'V',I2,',',I2,'=',F16.9)
 0017              15 FORMAT(1H1)
 0018              16 FORMAT(30X,'RESULTS OF PROFILE CORRELATION'/42X,'INPUT',21X,'PREDI
                     1CTED')
 0019              17 FORMAT(7X,'2X/A',10X,'2Y/B')
 0020              18 FORMAT(7X,'R/A',8X,'THETA/2PI')
 0021              19 FORMAT(2X,'CONCENTRATION',3X,'VELOCITY',4X,'CONCENTRATION',3X,'VE
                     1LOCITY')
 0022              20 FORMAT(3(2X,F12.5),14X,2(2X,F12.5))
 0023             201 FORMAT(2(2X,F12.5),14X,3(2X,F12.5))
 0024              21 FORMAT(5X,'STANDARD DEVIATION IN C=',E12.5,5X,'IN V=',E12.5)
 0025              22 FORMAT(30X,'AREA SEGMENT NUMBER')
 0026              23 FORMAT(2X,F12.5,2X,F12.5,7X,I3)
 0027              24 FORMAT(5X,'ERROR IN CVBAR=',4X,F12.5/5X,'ERROR IN CBAR*VBAR=',
                     *F12.5/5X,'ERROR IN VBAR=',4X,F12.5)
 0028              25 FORMAT(1X,'ERROR IN ENTERING CONCENTRATION PROFILE DATA.',5X,I3)
 0029              26 FORMAT(1X,'ERROR IN ENTERING VELOCITY PROFILE DATA.',5X,I3)
 0030              27 FORMAT(4I2)
 0031              28 FORMAT(1X,'PROBE',I3,'  LOCATED OUTSIDE DUCT BOUNDARIES.PROBLEM AB
                     *ORTED.')
 0032              29 FORMAT(5X,'PROBE',I3,'  ASSIGNED TO A NON-EXISTENT AREA SEGMENT.PR
                     *OBLEM ABORTED')
 0033              30 FORMAT(20X,'PROBE LOCATION DATA'/10X,'NUMBER OF PROBES=',I3)
 0034              31 FORMAT(1H0)
 0035              32 FORMAT(12X,'ACTUAL',41X,'COMPUTED'/4X,'AVERAGE V',6X,'AVERAGE C*V'
                     1,12X,'AVERAGE V',7X,'AVERAGE C*V',7X,'AVERAGE C')
 0036              33 FORMAT(1X,F12.3,5X,F12.3,10X,F12.3,5X,F12.3,5X,F12.3)
 0037            1401 FORMAT(20A4)
 0038            1403 FORMAT(//20X,20A4////)
```

```
FORTRAN IV G LEVEL  21                MAIN              DATE = 74065        13/06/37              PAGE 0003

 0039          2000 FORMAT(1X,10E13.6)
 0040          2001 FORMAT( ' INPUT AND OUTPUT ARRY OF CROUT ROUTIN FOR CONCENTRATION
                   *'/)
 0041          2002 FORMAT(1H1,' ROW DATA.'/' NO OF CON. DATA CARDS ',I3,5X,
                  * 'NO. OF VEL. DATA CARDS ',I3)
 0042          2003 FORMAT(//' INPUT AND OUTPUT ARRY OF CROUT ROUTIN FOR VEL.'//)
 0043          6000 FORMAT(' UNABLE TO SOLVL BY THIS METHOD PROBLEM ABORTED')
 0044            99 READ(5,1) NGC,A,B,NIC
 0045               IF(NGC-1) 100,101,152
 0046           100 I=0.
 0047               SN=1./A
 0048               SZ=1./6.2831853
 0049               GO TO 102
 0050           101 SN=2./A
 0051               SZ=2./B
 0052           102 READ(5,1401) COMENT
 0053                NARM=0
 0054               IF (NIC) 103,103,125
 0055           103 I=0
 0056          1121 I=I+1
 0057               READ(5,3) CON(I),XCRR(I),YORT(I),NN
 0058               IF(NN) 1122,1123,1122
 0059          1123 IF (CON(I)) 104,105,105
 0060           104 WRITE(6,25) I
 0061                NARM=1
 0062                GO TO 1121
 0063           105 IF (NCC) 106,106,110
 0064           106 IF (XCRR(I) - A) 107,107,104
 0065           107 IF (XORR(I)) 104,108,108
 0066           108 IF (YORT(I)-6.2831853)  109,109,104
 0067           109 IF (YORT(I))104,112,112
 0068           110 IF (ABS(XORR(I))-.5*A) 111,111,104
 0069           111 IF (ABS(YORT(I))-.5*B) 112,112,104
 0070           112 XORR(I)=SN*XORR(I)
 0071               YORT(I)=SZ*YORT(I)
 0072               GO TO 1121
 0073          1122 NCP=I-1
 0074               I=0
 0075          1201 I=I+1
 0076               READ(5,3) VEL(I),XRV(I),YTV(I) ,NN
 0077               IF(NN) 1124,1125,1124
 0078          1125 IF (NCC) 113,113,115
 0079           113 IF (XRV(I)-A) 115,115,114
 0080           114 WRITE(6,26)    I
 0081                NARM=1
 0082                GO TO 1201
 0083           115 IF (XRV(I)) 114,116,116
 0084           116 IF (YTV(I)-6.2831853) 117,117,114
```

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```
  117 IF (YTV(I)) 114,120,120
  116 IF (ABS(XRV(I))-.5*A) 119,119,114
  119 IF (ABS(YTV(I))-.5*B) 120,120,114
  120 XRV(I)=SN*XRV(I)
      YTV(I)=SZ*YTV(I)
      GO TO 1201
 1124 NVP=I-1
      READ(5,4) MCXOR,MCYOT,MVXOR,MVYOT
        IF(NARM) 1361,1202,1361
 1202 NV=2*MVXOR+3
      MV=2*MVYOT+3
      NC=2*MCXOR+3
      MC=2*MCYOT+3
      N=NV*MV
      IF (N-NVP) 122,122,121
  121 REL=NVP
      REL=SQRT(REL)
      NV=REL
      MV=NVP/NV
  122 N=NC*MC
      IF (N-NCP)124,124,123
  123 REL=NCP
      REL=SQRT(REL)
      NC=REL
      MC=NCP/NC
  124 MSIZE=NC*MC
      MSP1=MSIZE+1
      WRITE(6,2002) NCP,NVP
      CALL COEFG(NCP,CON,YORT,XCAR,          AR,MC,NC)
      WRITE(6,2001)
      WRITE(6,2000) ((AR(I,J),J=1,MSP1),I=1,MSIZE)
      WRITE(6,31)
      CALL CROUT(NARM1     ,AR,C,MC,NC)
      WRITE(6,2000) ((AR(I,J),J=1,MSP1),I=1,MSIZE)
      IF(NARM1) 1240,1241,1240
 1240 WRITE(6,6000)
      NARM=1
 1241 MSIZE=NV*MV
      MSP1=MSIZE+1
      CALL COEFG(NVP,VEL,YTV,XRV,          AR,MV,NV)
      WRITE(6,2003)
      WRITE(6,2000) ((AR(I,J),J=1,MSP1),I=1,MSIZE)
      WRITE(6,31)
      CALL CROUT(NARM2     ,AR,V,MV,NV)
      WRITE(6,2000) ((AR(I,J),J=1,MSP1),I=1,MSIZE)
       IF(NARM2) 1242,1243,1242
 1242 WRITE(6,6000)
      NARM=1
      1245 IF(NARM) 1361,126,1361
       125 READ(5,27) NC,MC,MV,NV
           READ(5,5) ((C(J,I),J=1,MC),I=1,NC)
           READ(5,5) ((V(J,I),J=1,MV),I=1,NV)
       126 WRITE(6,15)
           WRITE(6,1483) COMENT
           WRITE(6,8)
           IF (NGC) 127,127,128
       127 WRITE(6,9) A
           GO TO 129
       128 WRITE(6,10) A,B
       129 WRITE(6,11)
           WRITE(6,12)(((J,I,C(J,I)),J=1,MC),I=1,NC)
           WRITE(6,31)
           WRITE(6,13)
           WRITE(6,14)(((J,I,V(J,I)),J=1,MV),I=1,NV)
           WRITE(6,15)
           IF (MIC) 130,130,136
       130 WRITE(7,5) ((C(J,I),J=1,MC),I=1,NC)
           WRITE(7,5) ((V(J,I),J=1,MV),I=1,NV)
           ERC=0.
           ERV=0.
           WRITE(6,16)
           IF (NGC) 131,131,132
       131 WRITE(6,18)
           GO TO 133
       132 WRITE(6,17)
       133 WRITE(6,19)
           DO 134 I=1,NCP
           CVAL=POLGEN(C,MC,NC,XORR(I),YORT(I))
           ERC=ERC+(CVAL-CON(I))*(CVAL-CON(I))
           VVAL=POLGEN(V,MV,NV,XORR(I),YORT(I))
       134 WRITE(6,20) XORR(I),YORT(I),CON(I)     ,CVAL,VVAL
           WRITE(6,15)
           WRITE(6,16)
           IF (NGC) 1341,1341,1342
      1341 WRITE(6,18)
           GO TO 1343
      1342 WRITE(6,17)
      1343 WRITE(6,19)
           DO 135 I=1,NVP
           CVAL=POLGEN(C,MC,NC,XRV(I),YTV(I))
           VVAL=POLGEN(V,MV,NV,XRV(I),YTV(I))
           ERV=ERV+(VVAL-VEL(I))*(VVAL-VEL(I))
       135 WRITE(6,201) XRV(I),YTV(I)     ,VEL(I),CVAL,VVAL
           RNCP=NCP
           ERC=SQRT(ERC/RNCP)
           RNVP=NVP
```

```
0181            ERV=SQRT(ERV/ANVPB)
0182            WRITE(6,21) ERC,ERV
0183            WRITE(6,15)
0184        136 CALL CVINT(NGC,    C,NC,MC,V,NV,MV,CVI,VI)
0185       1361 READ(5,6) NST
0186            IF(NST) 99,99,1362
0187       1362 GO TO (138,137,137,137,137),NST
0188        137 CALL EXIT
0189        138 READ(5,1401) COMENT
0190            READ(5,2) NASEG
0191            NALARM=0
0192            I=0
0193       1392 I=I+1
0194            READ(5,7) XORR(I),YORT(I),IARSEG(I) ,NN
0195            IF(NN) 1390,1391,1390
0196       1391 IF (NGC) 139,139,144
0197        139 IF (XORR(I)-A) 141,141,140
0198        140 WRITE(6,28) I
0199            NALARM=1
0200            GO TO 1392
0201        141 IF (XORR(I)) 140,142,142
0202        142 IF (YORT(I)-6.2831653) 143,143,140
0203        143 IF (YORT(I)) 140,146,146
0204        144 IF (ABS(XORR(I))-.5*A) 145,145,140
0205        145 IF (ABS(YORT(I))-.5*B) 146,146,140
0206        146 IF (IARSEG(I)-NASEG) 148,148,147
0207        147 WRITE(6,29) I
0208            NALARM=1
0209            GO TO 1392
0210        148 IF(IARSEG(I)) 147,147,1481
0211       1481 XORR(I)=SN*XORR(I)
0212            YORT(I)=SZ*YORT(I)
0213            GO TO 1392
0214       1390 NPR=I-1
0215              IF(NALARM) 1361,1484,1361
0216       1484 IF(NARM) 1361,1485,1361
0217       1485  WRITE(6,1483) COMENT
0218            WRITE(6,20) NPR
0219            IF (NGC) 149,149,150
0220        149 WRITE(6,16)
0221            GO TO 151
0222        150 WRITE(6,17)
0223        151 WRITE(6,22)
0224            WRITE(6,23) (XORR(I),YORT(I),IARSEG(I),I=1,NPR)
0225            CALL PRSANS(IARSEG,XORR,YORT,NPR,      NASEG,C,MC,AC,V,MV,NV,
               1CBAR,VBAR,CVBAR)
0226            F1=ABS(1.-CVBAR/CVI)
0227            F2=ABS(1.-CBAR*VBAR/CVI)
```

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FORTRAN IV G LEVEL  21                MAIN               DATE = 74085        13/06/37           PAGE 0007

 0228             E3=ABS(1.-VBAR/VI)
 0229             WRITE(6,15)
 0230             WRITE(6,32)
 0231             WRITE(6,33) VI,CVI,VBAR,CVBAR,CBAR
 0232             WRITE(6,24) E1,E2,E3
 0233             WRITE(6,15)
 0234             GO TO 1361
 0235         152 CALL EXIT
 0236             END
```

```
FORTRAN IV G LEVEL  21                MAIN                DATE = 74085          13/06/37              PAGE 0008

                                   SUBPROGRAMS CALLED
SYMBOL     LOCATION      SYMBOL    LOCATION      SYMBOL    LOCATION      SYMBOL    LOCATION      SYMBOL    LOCATION
IBCOM#        348        COLFG        34C        CROUT        350        POLGEN       354        CVINT        358
EXIT          35C        PRLANS       360        SQRT         364

                                   SCALAR MAP
SYMBOL     LOCATION      SYMBOL    LOCATION      SYMBOL    LOCATION      SYMBOL    LOCATION      SYMBOL    LOCATION
NCC           4A8        A            4AC        B            4B0        NIC          4B4        SN           4B8
SZ            4BC        NARM         4C0        I            4C4        NN           4C8        NCP          4CC
EVP           4D0        MCXOR        4D4        MCYOT        4D8        MVXOR        4DC        MVYOT        4E0
EV            4E4        MV           4E8        NC           4EC        MC           4F0        N            4F4
ALL           4F8        MSIZE        4FC        MSP1         500        J            504        MARM1        508
MARM2         50C        EKC          510        FRV          514        CVAL         518        VVAL         51C
KNCP          520        RNVP         524        CVI          528        VI           52C        NST          530
NASEG         534        NALARM       538        NPR          53C        CBAR         540        VBAR         544
CVBAR         548        E1           54C        E2           550        E3           554

                                   ARRAY MAP
SYMBOL     LOCATION      SYMBOL    LOCATION      SYMBOL    LOCATION      SYMBOL    LOCATION      SYMBOL    LOCATION
AR            558        CON         EC00        VEL         F0B0        XORR        F560        YORT        FA10
YTV          FEC0        C          10370        V          10554        IARSEG     10738        XRV        10BE8
CUMENT      11098

                                   FORMAT STATEMENT MAP
SYMBOL     LOCATION      SYMBOL    LOCATION      SYMBOL    LOCATION      SYMBOL    LOCATION      SYMBOL    LOCATION
     1      110E6             2     110FA             3     110FE             4     11109             5     1110F
     6      11116             7     1111C             8     11128             9     1113F            10     1115D
    11      11167            12     11158            13     111CC            14     111E7            15     111F6
    16      11200            17     11238            18     1124D            19     11263            20     1129F
   201      112B3            21     112C7            22     112F4            23     1130D            24     1131D
    25      1136A            26     113A1            27     113D3            28     113D9            29     1141A
    30      11462            31     11493            32     11498            33     114F6          1401     11511
  1403      11517          2000     11525          2001     1152E          2002     1156C          2003     11586
  6000      115ED

*OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,MAP
*OPTIONS IN EFFECT*  NAME = MAIN    , LINECNT =       50
*STATISTICS*    SOURCE STATEMENTS =       236,PROGRAM SIZE =     76794
*STATISTICS*  NO DIAGNOSTICS GENERATED
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```
FORTRAN IV G LEVEL  21                 COEFG               DATE = 74085          13/06/37               PAGE 0001

                SUBROUTINE  COEFG(NP,Z,Y,X,          AR,M,N)
                DIMENSION Z(300),Y(300),X(300),AR(121,122)
                DO 600 I=1,N
                DO 500 J=1,M
                DO 300 IS=1,N
                DO 200 IR=1,M
                ICOL=(IS-1)*M+IR
                IROW=(I-1)*M+J
                AR(IROW,ICOL)=0.
                N1=IR+J-2
                N2 =I+IS-2
                DO 100 K=1,NP
                 POWER1=1.
                 POWER2=1.
                IF(N1) 1,1,2
              2 DO 3 L=1,N1
              3 POWER1=POWER1*Y(K)
              1 IF(N2) 4,4,5
              5 DO 6 L=1,N2
              6 POWER2=POWER2*X(K)
              4 AR(IROW,ICOL)=AR(IROW,ICOL)+POWER1*POWER2
            100 CONTINUE
            200 CONTINUE
            300 CONTINUE
                ICOL=ICOL+1
                AR(IROW,ICOL)=0.
                N1=J-1
                N2=I-1
                DO 400 K=1,NP
                POWER1=1.
                POWER2=1.
                 IF(N1) 10,10,11
             11 DO 12 L=1,N1
             12 POWER1=POWER1*Y(K)
             10 IF(N2) 13,13,14
             14 DO 15 L=1,N2
             15 POWER2=POWER2*X(K)
             13 AR(IROW,ICOL)=AR(IROW,ICOL)+Z(K)*POWER1*POWER2
            400 CONTINUE
            500 CONTINUE
            600 CONTINUE
                RETURN
                END
```

```
FORTRAN IV G LEVEL  21                COEFG              DATE = 74085        13/06/37            PAGE 0002

                             SCALAR MAP
SYMBOL    LOCATION     SYMBOL    LOCATION     SYMBOL    LOCATION     SYMBOL    LOCATION     SYMBOL    LOCATION
I               F8     N               FC     J              100     M              104     IS             108
IR             10C     ICOL           110     IROW           114     N1             118     N2             11C
K              120     NP             124     POWER1         128     POWER2         12C     L              130

                             ARRAY MAP
SYMBOL    LOCATION     SYMBOL    LOCATION     SYMBOL    LOCATION     SYMBOL    LOCATION     SYMBOL    LOCATION
Z              134     Y              138     X              13C     AR             140

 *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,MAP
 *OPTIONS IN EFFECT*  NAME = COEFG    , LINECNT =       50
 *STATISTICS*    SOURCE STATEMENTS =       43,PROGRAM SIZE =     1364
 *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
FORTRAN IV G LEVEL  21                  CROUT               DATE = 74065          13/06/37            PAGE 0001

      SUBROUTINE CROUT(NALARM ,AR,CORV,M,N)
        DIMENSION CORV(11,11),AR(121,122)
      NALARM=0
      TOL=1.E-9
      IA=M*N
      IAP1=IA+1
      DO 100 I=1,IA
      DO 100 J=1,IAP1
       SAVE=AR(I,J)
      IF(ABS(SAVE  )-TOL) 101,101,100
  101 AR(I,J)=0.
  100 CONTINUE
      IAM1=IA-1
      IF(AR(1,1)) 300,301,300
  301 J=1
      GO TO 209
  300 DO 1 J=2,IAP1
    1 AR(1,J)=AR(1,J)/AR(1,1)
      DO 6 MD=2,IA
      J=MD
      INDEX=MD-1
       AX=AR(J,J)
       DO 200 K=1,INDEX
  200 AX=AX-AR(J,K)*AR(K,J)
      IF(ABS(AX)-TOL) 201,201,202
  201 IF(J-IA) 209,205,205
  209 ND=J+1
      AMAX=0.
      IMAX=0
      DO 203 I=ND,IA
       SAVE=AR(I,J)
      IF(ABS(SAVE  ) -ABS(AMAX)) 203,203,204
  204 IMAX=I
      AMAX=AR(I,J)
  203 CONTINUE
        IF(IMAX) 205,205,206
  206 DO 207 I=1,IAP1
      SAVE=AR(J,I)
      AR(J,I)=AR(IMAX,I)
      AR(IMAX,I)=SAVE
  207 CONTINUE
      IF(J-1) 300,300,202
  205 NALARM=1
      RETURN
  202 DO 3 I=MD,IA
      DO 20 K=1,INDEX
      AR(I,J)=AR(I,J)-AR(I,K)*AR(K,J)
   20 CONTINUE
```

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FURTRAN IV G LEVEL  21                 CROUT              DATE = 74085          13/06/37              PAGE 0002

  0049              3 CONTINUE
  0050                I=MD
  0051                IS=MD+1
  0052                DO 5 J=IS,IAP1
  0053                DO 40 K=1,INDEX
  0054                AR(I,J)=AR(I,J)-AR(I,K)*AR(K,J)
  0055             40 CONTINUE
  0056                AR(I,J)=AR(I,J)/AR(I,I)
  0057              5 CONTINUE
  0058              6 CONTINUF
  0059                DO 8 I=1,IAM1
  0060                MD=IA-I
  0061                IS=MD+1
  0062                DO 70 K=IS,IA
  0063                AR(MD,IAP1)=AR(MD,IAP1)-AR(MD,K)*AR(K,IAP1)
  0064             70 CONTINUE
  0065              8 CONTINUE
  0066                DO 11 I=1,IA
  0067                MC=I-M*(I/M)
  0068                IF (MC) 9,9,10
  0069              9 MC=M
  0070                NC=I/M
  0071                GO TO 11
  0072             10 NC=I/M+1
  0073             11 CCRV(MC,NC)=AR(I,IAP1)
  0074                RETURN
  0075                END

FURTRAN IV G LEVEL  21                 CROUT              DATE = 74085          13/06/37              PAGE 0003

                                        SCALAR MAP
SYMBOL      LOCATION        SYMBOL    LOCATION        SYMBOL    LOCATION        SYMBOL    LOCATION        SYMBOL    LOCATION
NALARM         148          TOL          14C          IA           150          M            154          N            158
IAP1           15C          I            160          J            164          SAVE         168          IAM1         16C
MD             170          INDEX        174          AX           178          K            17C          ND           180
AMAX           184          IMAX         168          IS           18C          MC           190          NC           194

                                        ARRAY MAP
SYMBOL      LOCATION        SYMBOL    LOCATION        SYMBOL    LOCATION        SYMBOL    LOCATION        SYMBOL    LOCATION
CCRV           198          AR           19C

  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,MAP
  *OPTIONS IN EFFECT*  NAME = CROUT   , LINECNT =       50
  *STATISTICS*    SOURCE STATEMENTS =        75,PROGRAM SIZE =      2316
  *STATISTICS*  NO DIAGNOSTICS GENFRATED
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```
FORTRAN IV G LEVEL  21                CVINT              DATE = 74C85          13/06/37              PAGE 0001

           SUBROUTINE CVINT(NGC,    C,NC,MC,V,NV,MV,CVI,VI)
           DIMENSION C(11,11),V(11,11)
           CVI=0.
           VI=0.
           IF(NGC) 100,100,200
       100 DO 140 I=1,NC
           DO 130 J=1,MC
           DO 120 K=1,NV
           DO 110 L=1,MV
           D1=(J+L-1)*(I+K)
       110 CVI=CVI+C(J,I)*V(L,K)/D1
       120 CONTINUE
       130 CONTINUE
       140 CONTINUE
           CVI=2.*CVI
           DO 160 I=1,NV
           DO 150 J=1,MV
           D1=J*(I+1)
       150 VI=VI+V(J,I)/D1
       160 CONTINUE
           VI=2.*VI
           RETURN
       200 DO 240 I=1,NC
           DO 230 J=1,MC
           DO 220 K=1,NV
           DO 210 L=1,MV
           N= (J+L-1)*(I+K-1)
           D1=N
           IF (2*(N/2)-N) 205,210,205
       205 CVI=CVI+C(J,I)*V(L,K)/D1
       210 CONTINUE
       220 CONTINUE
       230 CONTINUE
       240 CONTINUE
           DO 260 I=1,NV
           DO 250 J=1,MV
           N=I*J
           D1=N
           IF (2*(N/2)-N) 245,250,245
       245 VI=VI+V(J,I)/D1
       250 CONTINUE
       260 CONTINUE
           RETURN
           END
```

```
FORTRAN IV G LEVEL  21                CVINT              DATE = 74085        13/06/37           PAGE 0002

                                  SCALAR MAP
SYMBOL      LOCATION        SYMBOL    LOCATION        SYMBOL    LOCATION        SYMBOL    LOCATION        SYMBOL    LOCATION
CVI            F0           VI            F4          MGC           F8          I             FC          NC           100
J             104           MC           108          K            10C          NV           110          L            114
MV            118           D1           11C          N            120

                                  ARRAY MAP
SYMBOL      LOCATION        SYMBOL    LOCATION        SYMBOL    LOCATION        SYMBOL    LOCATION        SYMBOL    LOCATION
C             124           V            128

   *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,MAP
   *OPTIONS IN EFFECT*  NAME = CVINT   , LINECNT =       50
   *STATISTICS*    SOURCE STATEMENTS =       44,PROGRAM SIZE =     1442
   *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
FORTRAN IV G LEVEL  21                PREANS             DATE = 74085         13/06/37              PAGE 0001

 0001            SUBROUTINE PREANS(IARSEG,XORR,YORT,NPR,      NASEG,C,MC,NC,V,MV,NV
                1     ,CBAR,VBAR,CVBAR)
 0002            DIMENSION IARSEG(300),XORR(300),YORT(300),IUSEG(300  ),C(11,11),
                1          V(11,11)
 0003            CBAR=0.
 0004            VBAR=0.
 0005            CVBAR=0.
 0006            DO 20 I=1,NASEG
 0007         20 IUSEG(I)=0
 0008            DO 40 I=1,NPR
 0009            J=IARSEG(I)
 0010         40 IUSEG(J)=IUSEG(J)+1
 0011            ICZ=0
 0012            DO 50 I=1,NASEG
 0013            IF (IUSEG(I)) 50,50,45
 0014         45 ICZ=ICZ+1
 0015         50 CONTINUE
 0016            DA=ICZ
 0017            DO 10 I=1,NPR
 0018            J=IARSEG(I)
 0019            F=IUSEG(J)
 0020            CB=POLGEN(C,MC,NC,XORR(I),YORT(I))
 0021            VB=POLGEN(V,MV,NV,XORR(I),YORT(I))
 0022            CBAR=CBAR+CB/F
 0023            VBAR=VBAR+VB/F
 0024         10 CVBAR=CVBAR+CB*VB/F
 0025            CBAR=CBAR/DA
 0026            VBAR=VBAR/DA
 0027            CVBAR=CVBAR/DA
 0028            RETURN
 0029            END
```

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```
FORTRAN IV G LEVEL  21              PRBANS              DATE = 74065        13/06/37            PAGE 0002

                                    SUBPROGRAMS CALLED
SYMBOL    LOCATION      SYMBOL    LOCATION      SYMBOL    LOCATION      SYMBOL    LOCATION      SYMBOL    LOCATION
FOLGIN       B0

                                    SCALAR MAP
SYMBOL    LOCATION      SYMBOL    LOCATION      SYMBOL    LOCATION      SYMBOL    LOCATION      SYMBOL    LOCATION
CBAR         DC         VBAR         E0         CVBAR        E4         I            F8         NASEG        FC
NPN          F0         J            F4         ICZ          F8         DA           FC         F            100
CC           104        MC           108        NC           10C        Vb           110        MV           114
NV           118

                                    ARRAY MAP
SYMBOL    LOCATION      SYMBOL    LOCATION      SYMBOL    LOCATION      SYMBOL    LOCATION      SYMBOL    LOCATION
IANSEG       11C        XORK         120        YORT         124        IUSFG        128        C            5D8
V            5DC

  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,MAP
  *OPTIONS IN EFFECT*  NAME = PRBANS  , LINECNT =       50
  *STATISTICS*    SOURCE STATEMENTS =       29,PROGRAM SIZE =      2388
  *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
FORTRAN IV G LEVEL  21              POLGEN           DATE = 74065       13/06/37           PAGE 0001

                    FUNCTION POLGEN(CORV,M,N,X,Y)
                    DIMENSION CORV(11,11)
                    POLGEN=0.
                    DO 2 I=1,N
                    PG=0.
                    N1=I-1
                    DO 1 J=1,M
                    N2=J-1
                    POWER=1.
                    IF(N2) 1,1,3
                  3 DO 4 L=1,N2
                  4 POWER=POWER*Y
                  1 PG=PG+CORV(J,I)*POWER
                    POWER=1.
                    IF(N1) 2,2,5
                  5 DO 6 L=1,N1
                  6 POWER=POWER*X
                  2 POLGEN=POLGEN+PG*POWER
                    RETURN
                    END
```

```
FORTRAN IV G LEVEL  21              POLGEN            DATE = 74085        13/06/37          PAGE 0002

                                  EQUIVALENCE DATA MAP
SYMBOL     LOCATION      SYMBOL     LOCATION      SYMBOL     LOCATION      SYMBOL     LOCATION      SYMBOL     LOCATION
POLGEN         C0

                                  SCALAR MAP
SYMBOL     LOCATION      SYMBOL     LOCATION      SYMBOL     LOCATION      SYMBOL     LOCATION      SYMBOL     LOCATION
I              C4        N              C8        PG             CC        N1             D0        J              D4
M              D8        N2             DC        POWER          E0        L              E4        Y              E8
X              EC

                                  ARRAY MAP
SYMBOL     LOCATION      SYMBOL     LOCATION      SYMBOL     LOCATION      SYMBOL     LOCATION      SYMBOL     LOCATION
CORV           F0

  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,MAP
  *OPTIONS IN EFFECT*  NAME = POLGEN  , LINECNT =       50
  *STATISTICS*    SOURCF STATEMENTS =        20,PROGRAM SIZE =       732
  *STATISTICS*  NO DIAGNOSTICS GENERATED

  *STATISTICS*  NO DIAGNOSTICS THIS STEP
```

```
                                   OS/360 LOADER

OPTIONS USED - PRINT,MAP,LET,CALL,NORES,NOTERM,SIZE=100224,NAME=**GO

   NAME   TYPE  ADDR        NAME   TYPE  ADDR        NAME   TYPE  ADDR        NAME   TYPE  ADDR        NAME   TYPE  ADDR

 MAIN       SD 124810     CDEFG      SD 137410     CROUT      SD 137968     CVINT      SD 138278     PARAMS     SD 138820
 PULGEN     SD 139178     IHCSSQRT*  SD 139458     SQRT     * LR 139458     IHCFEXIT*  SD 1395A0     EXIT     * LR 1395A0
 IHCECOMH*  SD 1395C0     IBCOM#   * LR 1395C0     FDIOCS#  * LR 13967C     INTSWTCH*  LR 13A504     IHCCOMH2*  SD 13A528
 SEQDASD *  LR 13A0A0     IHCFRRM  * SD 13A600     FRAXON   * LR 13AF86     IHCERRE  * LR 13A6A0     IHCUOPT  * SD 13B160
 IHCEFNTH*  SD 13B4B0     ARITH#   * LR 13B460     ADJSWTCH*  LR 13B64C     IHCEFIOS*  SD 13B9F8     FIOCS#   * LR 13B9F8
 FIOCSBEP*  LR 13B9FE     IHCFIOS2*  SD 13C920     IHCFCVTH*  SD 13CE90     ADCON#   * LR 13CE50     FCVAOUTP*  LR 13CEFA
 FCVLOUTP*  LR 13CF0A     FCVZOUTP*  LR 13D00A     FCVIOUTP*  LR 13D488     FCVLOUTP*  LR 13D90A     FCVCOUTP*  LR 13D8A6
 INT6SWCH*  LR 13DE88     IHCUATBL*  SD 13DFF0     IHCETRCH*  SD 13E620     IHCTRCH  * LR 13E628     ERRTRA   * LR 13E630

 TOTAL LENGTH    1A0A8
 ENTRY ADDRESS  124810
```